Equilibrium Sorting and the Gender Wage Gap

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April 17, 2026†

Abstract

Women face a "glass ceiling" in career advancement, with considerable underrepresentation in top positions. This may arise because employers anticipating more career interruptions from women might allocate them to low-productivity jobs. I develop an equilibrium search model to quantify mechanisms underlying the life-cycle gender wage gap: workers' skill accumulation, amenity preferences, and employers' wage-setting and job allocation decisions. Estimating the model on administrative data from Finland, I find that employer responses account for a sizable share of the gender wage gap in early career, while differences in labor force attachment are a major factor in late career. Policy counterfactuals show that a more equal division of parental leave is most effective at reducing gender gaps both before and after childbirth. Requiring equality on either the wage or hiring margin induces firms to adjust along the other margin.

JEL-codes: J23, J64, J24, J16, E24, J32
Keywords: Gender wage gap, equilibrium search model, sorting, human capital, non-wage amenities

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†First version: November 16, 2019.

1 Introduction

The gender wage gap expands substantially over the life-cycle, as women advance much more slowly in their careers than their male counterparts. A large proportion of this gap can be attributed to men and women sorting into different industries, firms, and occupations.[1] Women remain heavily underrepresented in high-earning, high-status occupations, and a "glass ceiling" is documented in many countries.[2] However, less is known about the extent to which this differential sorting is driven by *workers' preferences* over job attributes or *employers' decisions* to assign men and women to different positions. Since match formation and wages are influenced by both workers and firms in the labor market, it is important to consider both labor supply and demand sides when designing policies aimed at reducing gender inequality. On one hand, we have policies to bolster women's labor supply, their stable employment after childbirth and access to top-level jobs; but on the other hand, these same policies may trigger employer responses along wage-setting and hiring margins, potentially offsetting their intended effects.

Using administrative matched employer-employee data from Finland, I first document gender gaps in labor market outcomes for university graduates. Over the life-cycle, the unconditional gap between men and women's mean hourly wages increases rapidly from 12 log points at labor market entry to 20 log points after ten years. A "glass ceiling" in women's career starts to emerge six years after labor market entry, as the most successful women fail to move into high-paid managerial positions. This widening gap coincides with family formation, which plays a central role in shaping subsequent career paths. While female and male university graduates have similar employment rates before having children, their trajectories diverge markedly after childbirth. Mothers take on average 18 months of parental leave per child, compared to about 2 months for fathers. Following parental leave, women's employment remains more precarious, as they are more than twice as likely as men to transition from employment to non-employment. They are also more likely to reduce hours, switch to part-time work, and move to jobs with better amenities (in terms of a family-friendly index) after childbirth.

Similar gender gaps in labor market behaviors and wage outcomes are documented in other high-income countries, and a large literature on "motherhood penalty" has highlighted the impact of children on women's careers.[3] However, this literature is relatively

[1]See Blau and Kahn (2017) and Altonji and Blank (1999) for comprehensive reviews.

[2]For example, in Sweden (Albrecht, Bjorklund and Vroman, 2003), the US (Matsa and Miller, 2011), Norway (Bertrand, Black, Jensen and Lleras-Muney, 2018), and many developing countries (Pande and Ford, 2012).

[3]For an overview of "motherhood penalties" across countries, see Kleven, Landais, Posch, Steinhauer

silent on the mechanisms behind the motherhood penalty and the gender gaps that already exist before childbirth. Identifying the sources of these gaps—both before and after children arrive—is crucial for policy design. If they largely reflect differences in accumulated experience, then policies that support labor supply (such as parental leave and childcare provision) may be most effective. If instead a substantial share of the gaps arises from firms' compensation and promotion decisions, interventions targeting workplace practices become more relevant. In practice, multiple forces are likely at play, and their relative importance may vary over the life-cycle, potentially reinforcing one another.

Motivated by the empirical observations, this paper examines three leading explanations for the diverging career paths of men and women over the life-cycle. First, the *human capital* channel: work interruptions after having children hinder women's experience accumulation on the job, so they progress more slowly than their male colleagues. Second, the *amenity preference* channel: women may sort into jobs that pay lower wages but offer more flexibility and other family-friendly amenities.[4] Third, the *firm response* channel: employers might anticipate women to have more fertility-related separations and absence that are costly for the firm, so they allocate different jobs to men and women (e.g. placing women on "mommy tracks"), or offer different wages to male and female workers for the same job, or both.[5]

In order to quantify the relative importance of the above channels, I develop an equilibrium search and matching model that features human capital dynamics, two-dimensional heterogeneity in productivity and amenity, and employer-side mechanisms. In the model, workers gain skills and conduct on-the-job search while employed, but human capital stagnates while unemployed or on parental leave. Male and female workers compete for the same jobs in the same labor market, but can differ in turnover rates, childcare responsibilities, and preferences for family-friendly amenities. Frictions in the labor market imply that it takes time for workers to find jobs and for vacancies to find workers.[6]

To investigate sorting, the model incorporates a fundamental scarcity of jobs similar to

and Zweimüller (2019) and Kleven, Landais and Leite-Mariante (2024).

[4]Felfe (2012), Goldin (2014) and Wiswall and Zafar (2017) show that women sort into occupations with temporal flexibility and fewer working hours. Adda, Dustmann and Stevens (2017) and Hotz, Johansson and Karimi (2018) find long-term career consequences for women working in family-friendly occupations.

[5]Although employers' expectations might be correct on average and their decisions are rational, such differential wage-setting and job allocation practices towards men and women would constitute the notion of statistical discrimination as in Arrow (1972) and Phelps (1972) – employers cannot observe the individual's labor force attachment, so they make decisions for each individual worker based on his/her group average characteristics.

[6]This paper remains agnostic about the exact source of frictional costs. Conceptually, the cost of a job vacancy in this paper might include the cost of searching, recruiting or training new workers, reorganizing the workforce, paying overtime hours, etc. in addition to the opportunity cost of forgone production.

Shimer and Smith (2000). This scarcity creates a unique dynamic where employers must carefully weigh the trade-offs between hiring men and women, considering the option values of keeping the vacancy open and waiting for potentially better matches. Hiring a woman may result in a lower match surplus due to the costs associated with higher expected turnovers. Therefore, differential sorting by gender across occupations and firms is not only driven by workers' preferences, but also production complementarities and employers' cost concerns.

In a capacity-constrained environment, highly productive jobs (such as managerial positions) may set higher qualification thresholds for employing women than men. This occurs because vacancy values are positive and increasing along the job productivity ladder, and top positions have more to lose when a worker departs. Since top managerial jobs have very high outside option values, the expected future production values with women may not be sufficiently large to make the match worthwhile. This leads to the emergence of a "glass ceiling" effect, where high-end jobs have a lower probability of matching with women than men of the same productivity. I show that a conventional job ladder model without capacity constraints cannot generate the "glass ceiling" phenomenon evident in empirical observations.

The model not only allows me to quantify the mechanisms underlying the large "motherhood penalties" after childbirth, but also helps to unpack gender gaps that already exist before childbirth. By combining three prominent explanations into one unified framework, I use the model to examine any interactions between the channels and assess the effectiveness of potential policy interventions. I also estimate a version of the model with endogenous fertility, and find that it does not match the observed fertility patterns in the data. I discuss the implications for the counterfactual results in Appendix K.

Using the method of simulated moments, I estimate the baseline model on the Finnish matched employer-employee data combined with Quality of Work Life surveys on job amenities. Identification exploits several sources of variation in the data. First, life-cycle variation in fertility risk helps distinguish firms' responses from taste-based discrimination: in the non-fecund period, any residual gender wage gap, conditional on accumulated human capital and job type, cannot be attributed to statistical discrimination based on child-related concerns. Second, the QWL survey data provide direct measures of job amenities, so amenity types can be treated as observed and sorting patterns identify workers' preferences. Finally, the linked employer–employee structure of the data, together with observed mobility patterns, gender composition within jobs, and wage dynamics over the life cycle, identifies human capital accumulation and production parameters separately.

The model accurately fits key moments of the data. The CES production function estimates show strong complementarity between workers' skills and job productivity, leading to positive assortative matching (PAM) for both men and women. Human capital growth estimates are higher in more productive jobs. This implies that even though men and women face the same learning rates for a given job, their human capital stocks can still diverge over the life-cycle as they sort into different jobs and work for different amounts of time. Prior to parenthood, estimates for male and female preferences are similar; however, women's valuation of family-friendly amenities increases substantially following childbirth. Women's separation rates are higher than men's, especially after having children.

Given the parameter estimates, I first decompose the life-cycle gender wage gap. The model reveals that 4 log points (out of 9) of the gender wage gap during the initial stages of a career can be attributed to employers' differential sorting and wage-setting by gender based on fertility-related concerns. As workers progress beyond child-rearing ages, the influence of employers' gender-specific decisions diminishes. Instead, a significant majority (11 out of 15 log points) of the wage gap in later career stages can be ascribed to gender disparities in labor force attachment and the cumulative deficit in women's human capital. Amenity preferences affect sorting patterns primarily after childbirth and are responsible for about 1.5 log points of the overall wage gap over the life-cycle. The residual wage gap, which could be due to employers' taste-based discrimination or initial productivity differences between men and women, accounts for approximately 17% of the total gap.

I also use the model to decompose the forces underlying women's underrepresentation in top positions. In the status quo economy, women face fewer opportunities than men to access high-productivity jobs prior to childbirth, while gender differences in job allocation disappear after the fertile period. In a counterfactual where employers anticipate identical parental leave and separation rates across genders, women gain greater access to high-productivity jobs and reject more low-productivity jobs early in their careers. As a result, the share of women in top positions rises from 38 to 43 percent in the six years before childbirth. Since high-productivity jobs provide more learning opportunities, the human capital channel interacts with the equilibrium sorting channel and amplifies the effects of job access. Enhanced skill levels make women less likely to endogenously exit employment after childbirth. The improved job allocation therefore has persistent effects, raising the female share in top positions by 4 to 2 percentage points between five and fifteen years after childbirth.

The model also provides insights into the efficiency-equity trade-offs of potential policies. I simulate the effects of three policy interventions. First, a "daddy months" expansion that shifts two months of parental leave from women to men closes the wage gap by 13% throughout the life-cycle. It incurs a small welfare loss of 0.02% because men are already at more advanced positions than women by the time they have children, so the production loss from men staying home for two months is less than compensated by women working two months more. Second, an equal hiring policy in top jobs improves women's representation in managerial positions, but employers undo this policy by exerting more wage discounts. Moreover, the increased access to top jobs is short-lived as the policy does not address the negative impacts of motherhood. However, the human capital gains in high-quality jobs have positive externalities, which increase social welfare by 0.01%. Third, an equal pay counterfactual shows that requiring firms to pay the same wage to similar men and women closes the gender wage gap by 15% on average. However, this requirement has unintended consequences as employers adjust on the job allocation margin. Women are more likely to be unemployed, and the proportion of women in top jobs decreases by 1 percentage point twenty years after labor market entry.

Overall, the counterfactual results suggest that achieving gender equality at the workplace is difficult without greater equality in family responsibilities, such as sharing parental leave more equally between men and women. The challenge arises from two key factors. First, strong positive assortative matching in the high-skill labor market pushes women toward lower-tier jobs when they accumulate less experience. Second, equilibrium labor demand responses imply that employers will continue to offer women lower wages and fewer high-level positions if they expect women to be the primary caregivers.

The results in this paper contribute both theoretically and empirically to the literature studying gender differences in job search and career outcomes. First and foremost, the paper adds to the search model literature that emphasize gender differences in separation risks and mobility (Bowlus, 1997; Liu, 2016; Gray, 2021; Bartolucci, 2013; Amano, Baron and Xiao, 2024), taste-based discrimination (Flabbi, 2010), amenity preferences (Le Barbanchon, Rathelot and Roulet, 2020; Morchio and Moser, 2026; Faberman, Mueller and Şahin, 2026), and personality traits (Flinn, Todd and Zhang, 2024). A common feature of this literature is its focus on the *cross-sectional* gender wage gap. For example, Flinn, Todd and Zhang (2024) finds that gender differences in agreeableness and emotional stability account for 10-12% of the gender wage gap by reducing women's bargaining power. Morchio and Moser (2026) and Faberman, Mueller and Şahin (2026) decompose the cross-sectional gender wage gap into amenity valuations and job ladder effects, and find that non-wage amenities explain a substantial share.

This paper instead focuses on the *life-cycle evolution* of gender disparities. Like Amano, Baron and Xiao (2024), I incorporate human capital dynamics and life stages such as childbirth and parental leave, allowing gender gaps to be decomposed at different points of a career. This paper departs from Amano, Baron and Xiao (2024) in two key ways. First, I introduce two-dimensional sorting across jobs that differ in both productivity and family-friendly amenities. Second, I model a common labor market for men and women in which firms decide on both wages and hiring thresholds, introducing a novel job allocation margin. Even among jobs offering similar amenities, high-productivity positions can impose higher hiring thresholds for women as employers anticipate career interruptions. As a result, women may sort into lower-productivity jobs for reasons unrelated to amenity preferences, providing a more comprehensive characterization of equilibrium sorting. This margin is particularly important in a life-cycle setting, as restricted access early in a career hinders human capital accumulation and affects subsequent job choices. Policy counterfactuals highlight the importance of modeling both wage and job allocation margins, as interventions targeting one margin may generate unintended consequences on the other margin.

The model in this paper is built on a body of search-matching literature with wage bargaining (Dey and Flinn, 2005; Cahuc, Postel-Vinay and Robin, 2006), human capital accumulation (Burdett, Carrillo-Tudela and Coles, 2011; Bagger, Fontaine, Postel-Vinay and Robin, 2014; Lise and Postel-Vinay, 2020; Herkenhoff, Lise, Menzio and Phillips, 2024), and job amenities (Dey and Flinn, 2008; Taber and Vejlin, 2020). Importantly, sorting of workers to jobs depends on match complementarities in the production function,[7] and I estimate a flexible CES production function rather than a multiplicative or Cobb-Douglas function commonly used in the search literature. By integrating several key modeling components into a single framework, my paper not only quantifies the various factors contributing to gender gaps, but also explores complex interactions between the channels that individual studies may overlook.

Finally, this paper relates to the theoretical literature on the gendered division of home responsibilities and firms' responses to this gender asymmetry. Hancock, Lafortune and Low (2025) shows that housework allocations deviate from standard comparative advan-

[7]See for example two-sided matching market (Becker, 1973; Shimer and Smith, 2000; Eeckhout and Kircher, 2011), with on-the-job search (Lise, Meghir and Robin, 2016; Hagedorn, Law and Manovskii, 2017; Lopes de Melo, 2018; Borovickova and Shimer, 2024), and with endogenous search effort (Bagger and Lentz, 2018).

tage predictions: even female breadwinners perform more chores than their male partners. Calvo, Lindenlaub and Reynoso (2024) highlights how the division of home and market hours shapes equilibrium labor market sorting and the gender pay gap through its effects on spouses' labor supply. Erosa, Fuster and Restuccia (2016) and Adda, Dustmann and Stevens (2017) develop dynamic models of human capital accumulation, fertility, and female labor supply to quantify the impact of children on the gender wage gap. Complementing this literature, my paper takes the household division of labor as given and focuses instead on employers' equilibrium responses to women's role in the family, both before and after childbirth. This approach captures reallocation across jobs and wages that partial equilibrium models may miss.

There are several approaches to model firms' statistical discrimination when facing gender differences in labor market attachment. Under asymmetric information, firms offer women lower wages (Albanesi and Olivetti, 2009; Gayle and Golan, 2012) or invest less in their training and promotion (Thomas, 2024); under symmetric information, Barron, Black and Loewenstein (1993) shows that similar sorting arises when higher female quit rates make training less profitable. Bronson and Thoursie (2021) documents that employers pass over women for promotion in anticipation of future labor supply reductions, even for women who never have children. Almar, Friedrich, Reynoso, Schulz and Vejlin (2025) introduces marital sorting and embeds family-firm interactions in a life-cycle model. One major advantage of adopting a frictional job search framework in my paper is tractability. It accommodates granular job types (occupations within firms) and many worker types, thereby generating a clear notion of career paths or "job ladders" over the life-cycle. The rich heterogeneity in job types allows me to analyze gender disparities in career advancement across the job productivity distribution, and quantify any welfare losses resulting from worker-job mismatches at different stages of a career.

2 Data

The Finnish Longitudinal Employer-Employee Data (FOLK) provides information on workers' demographics, monthly employment histories, children's birth dates and parental leave claims for the full population between years 1988 and 2016. The Structure of Earnings Statistics (SES) 1995-2013 contains full-time equivalent hourly wages, part-time status, contracted hours and 4-digit occupation codes. Such detailed occupation codes are typically not available from tax registers.[8]

[8]The SES covers 55 to 75 percent of private sector workers depending on the year, and under-samples small firms. Since I do not include small firms with 2 workers or less, data coverage is not a big issue. In the estimation, I use sample weights to account for potential missing data from small firms.

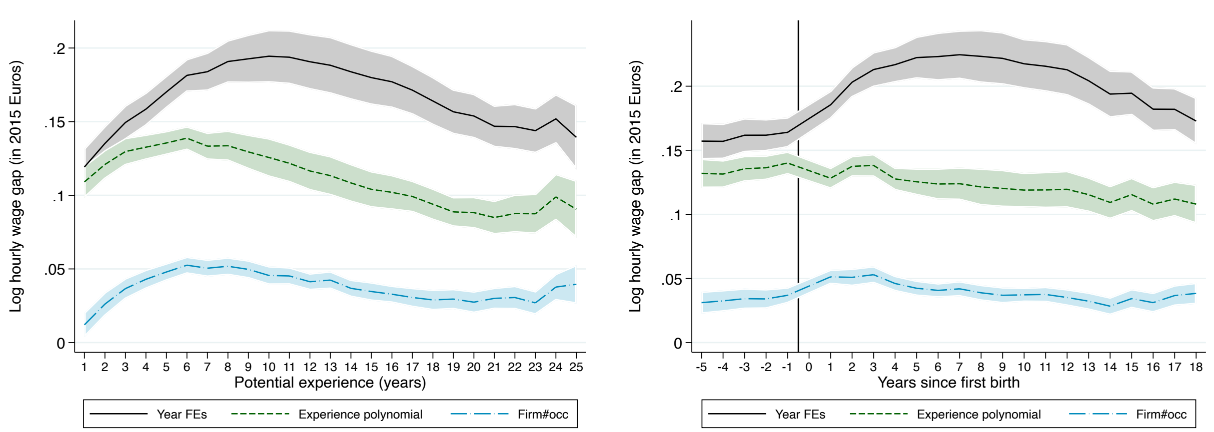
Since educated workers experience the largest increase in the gender wage gap over the life-cycle, in this paper I will focus on individuals who obtained master's degrees[9] in the years 1988 to 2005 so that we observe at least 8 years of labor market activities. Appendix A provides more details on sample restrictions.

2.1 Descriptive decomposition of the gender wage gap

To study the evolution of the gender wage gap over the life-cycle, I begin with a descriptive decomposition that sequentially adds controls. Figure 1(A) plots the log hourly wage gap by years since graduation (potential experience): *(i)* unadjusted (year fixed effects only); *(ii)* adding a quadratic in actual experience;[10] and *(iii)* further adding a full set of interactions between 4-digit occupation and firm fixed effects.

FIGURE 1. Descriptive decomposition of the gender wage gap

(A) Over the life-cycle (B) Around childbirth



NOTES: The lines represent the coefficients on the male dummy interacted with potential experience or years since childbirth. Shaded areas represent 95% confidence intervals. Standard errors are clustered at the firm level. The coefficients are obtained from regressions of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience in addition to (i); (iii) a full set of interactions of firm and occupation dummies in addition to (i) and (ii).

The unadjusted gap in Figure 1(A) rises from 12 log points at labor market entry to 20 log points after 10 years, before declining gradually to 15 log points after 25 years (when workers are above age 50). As women spend more time out of employment especially after childbirth, differences in actual experience explain an increasing share of the gap over the life-cycle. An "unexplained" gap (the bottom blue line in Figure 1(A)) of

[9]Master's degree in Finland is roughly equivalent to US bachelor's degree, since Finnish students who get into academic-track bachelor's programs are automatically enrolled in the master's programs.

[10]Actual experience is defined as cumulative months worked after university graduation. Since both men and women may work before completing a master's degree, I measure experience from bachelor's graduation, excluding short-term jobs (3 months or less) and summer internships. By master's graduation, men have 1.9 years of experience and women 1.6 years; the difference is not statistically significant.

about 4 log points still remains after controlling for occupations, firms and their interactions, suggesting that men and women are offered different wages even when they have the same actual experience and work in the same job within the same firm. Appendix Figure B1 shows similar results after adding additional controls for the field of study in university.[11] The existence of "unexplained" gaps might imply unequal pay for similarly qualified workers, potentially due to statistical or taste-based discrimination.

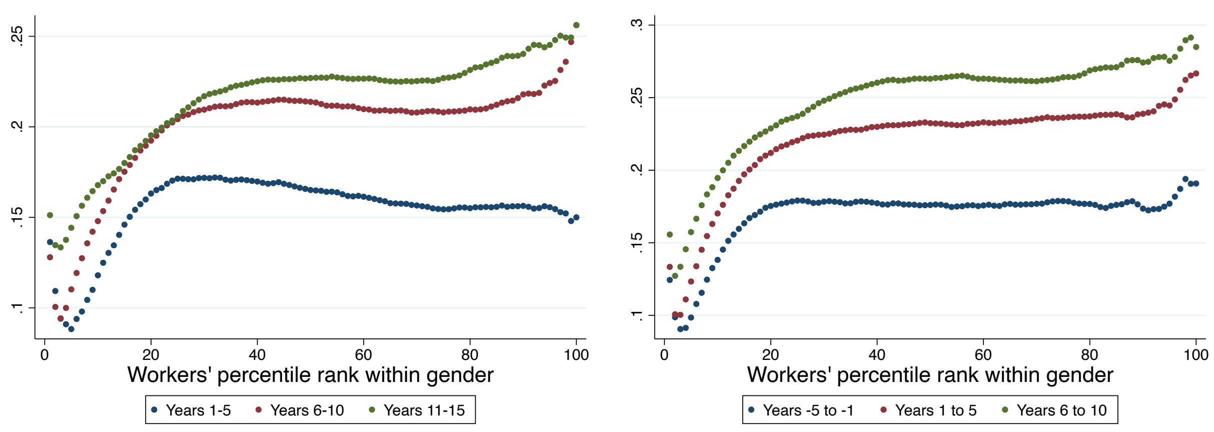
To assess the role of children, I perform a similar decomposition around the birth of the first child. Figure 1(B) shows that a gap of 14 log points already exists prior to childbirth and widens to 21 log points seven years after. Notably, the "unexplained" component is present even before birth, potentially suggesting that firms may anticipate fertility and treat men and women differently even before they have children.

Together, Figure 1(A) and Figure 1(B) document a widening gender wage gap over the life-cycle. While much of the gap reflects gender differences in accumulated experience, sorting across occupations and firms also plays an important role. The within-job "unexplained" component is small but statistically significant both before and after childbirth. However, these descriptive patterns should be interpreted with caution. The unexplained gap may understate discrimination, as experience, occupations and firms are themselves endogenous to earlier disparities. This motivates the structural model developed in Section 3 to quantify the drivers of differential sorting and divergent wage paths by gender.

FIGURE 2. Gender wage gap across the distribution

(A) Over the life-cycle

(B) Around childbirth



NOTES: Each dot represents the log wage difference between men and women of the same percentile rank in their respective distribution. Panel (A) shows the gender wage gap in each percentile of the distribution by the number of years since graduation, in 5-year windows; panel (B) shows the same statistics during the 5 years before childbirth, and 5 and 10 years post-birth.

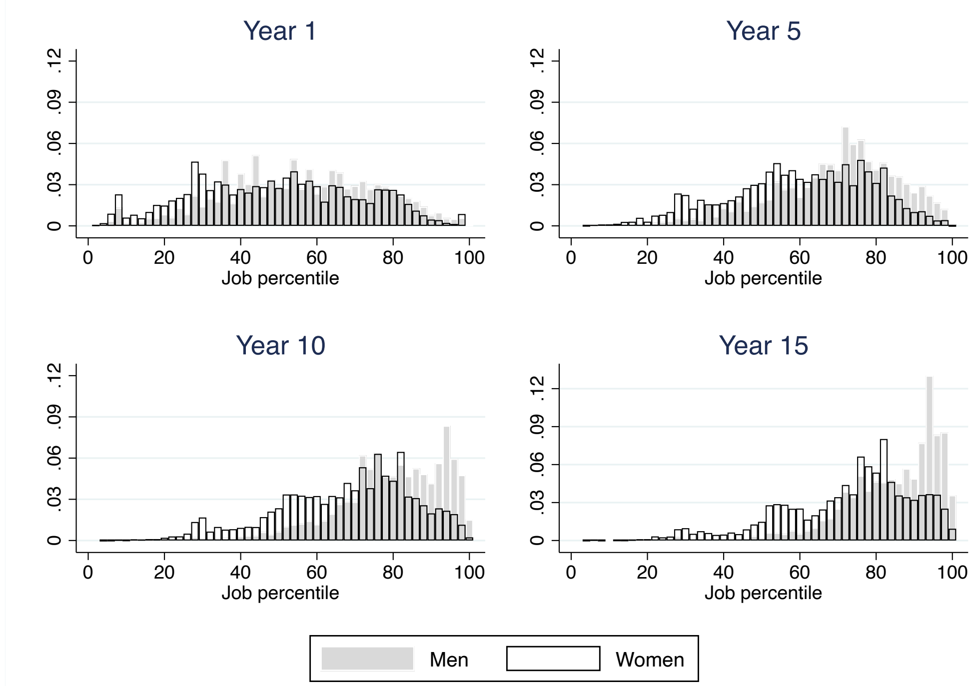
[11]Detailed field of study adds little explanatory power once occupation is controlled for, given their high correlation.

2.2 A glass ceiling to women's career advancement

Gender differences in career trajectories are evident not only in mean wages but across the entire wage distribution. The following figures plot the gender wage gap at each percentile.

Figure 2(A) shows that gaps at all percentiles widen with potential experience, with a glass ceiling emerging 6-10 years after labor market entry. Early in the career (years 1-5), the gap is similar across the upper distribution (around 16 log points above the 75th percentile). By years 6-10, however, the gap at the very top widens substantially (25 log points), exceeding that at the 75th percentile (21 log points), suggesting that top women face barriers to reaching the highest-paying positions. As shown in Figure 2(B), this glass ceiling is present even before the first birth, though it becomes more pronounced afterward.[12]

FIGURE 3. Sorting of men and women by years since labor market entry



NOTES: The histograms represent the distributions of men and women in the top quartile across all jobs in the economy. The x-axis is the job percentile, where jobs are ranked by their long-term average wage from 0 (lowest-paid) to 100 (highest-paid). Each job is a four-digit occupation within a firm.

To understand the career paths of women most affected by the glass ceiling, I examine the sorting patterns of top men and women over the life-cycle. Figure 3 tracks the mobility of top-quartile earners (those in the top 25% of their respective wage distribution for at

[12] Albrecht, Bjorklund and Vroman (2003) and Albrecht, Thoursie and Vroman (2015) document similar patterns in Sweden.

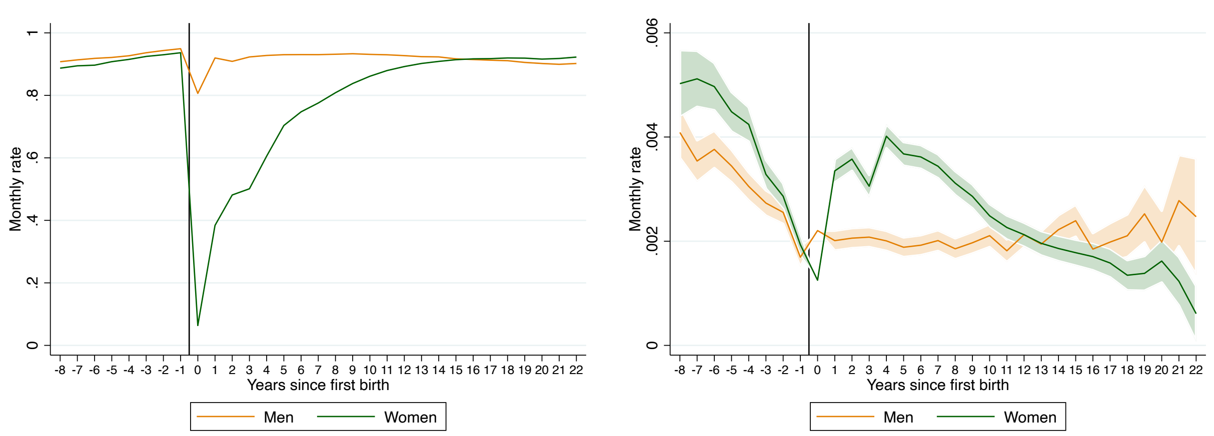
least 3 years) across firm-occupation cells. While men and women sort into similar jobs at labor market entry, men move into higher-ranked jobs more quickly. By year 10, there is already a concentration of men in the highest-paid jobs. From years 10 to 15, women barely enter into jobs beyond the 90th percentile (which are mainly managerial positions), whereas many men do.[13]

2.3 Labor force attachment

At least part of the gaps in wages and career advancement between men and women might be attributed to gender differences in labor force attachment. The Finnish parental leave system is very generous (see Appendix D for a detailed description). Master's graduated women take on average 18 months of paid leave for each child compared to only 2 months taken by men with master's degree.[14]

FIGURE 4. Labor force attachment around childbirth

(A) Employment rate (PL=not working) (B) E-to-U rates



NOTES: The lines represent the regression coefficients on the number of years since first birth, with calendar year fixed effects. The outcome variables are: (A) employment indicators at a monthly level; and (B) indicators of E-to-U transitions at a monthly level, for those who are not in parental leave. Shaded areas represent 95% confidence intervals.

Figure 4(A) shows that men and women have similar employment rates before having children, but labor supply diverges drastically at childbirth: virtually all women take leave, and the female employment rate only fully recovers some 14 years after the first birth. This divergence is compounded by a spike in women's E-to-U separation rate that remains well above men's for nine years post-birth (Figure 4(B)). Both channels impede

[13]The jobs beyond 90th percentile consist of business/finance/R&D managers (73%), medical doctors (9%), lawyers (3%), and business professional and engineer occupations at high-paid firms (14%).

[14]Parental leave duration is inferred from the annual parental leave allowance and home care allowance claims according to a schedule detailed in Appendix D.

career progression: human capital stagnates during non-employment, and repeated separations cost women their position on the job ladder, forgoing the rents and upward mobility of on-the-job search.

Women who return to work after childbirth also reduce their hours: the share working part-time rises from 5 to 15 percent in the year after birth and remains elevated for a decade (Figure B2). Since part-time arrangements are not always available at the same job, 58 percent of women who switch to part-time must change firms or occupations to do so (Altonji and Paxson, 1992). This partly contributes to the post-birth shift toward higher-amenity jobs documented in Section 2.4.

2.4 Family-friendly amenities

Workers' job choices can be driven by both wages and non-wage amenities e.g. job hazards, working conditions, stress and well-being and so on. In this paper, I focus only on the job amenities that are documented to be valued differentially by men and women, such as reduced hours and flexible work schedules (Goldin and Katz, 2011; Flabbi and Moro, 2012; Felfe, 2012; Goldin, 2014; Edwards, 2014; Wiswall and Zafar, 2017).[15]

I construct a family-friendly amenity index for each job that captures two dimensions: flexibility (e.g. the ability to work from home, adjust start/finish times etc.) and overwork demands (e.g. uncompensated overtime and contact outside working hours). These survey-based measures are combined with actual hours worked and the prevalence of part-time work in each firm-occupation cell, and the index is computed as the first principal component of all variables. Details are provided in Appendix C.

Using this amenity index, several interesting patterns emerge. First, jobs with high amenity index values are more abundant in the middle and lower end of the job productivity distribution (see Figure 5). Second, workers move from high- to low-amenity jobs over the life-cycle, because jobs become more demanding and require more hours and overtime as people climb the career ladder (e.g. to managerial positions). Third, Figure C1 shows that women are in jobs with slightly higher amenity index than men before childbirth, but there is a remarkable divergence after childbirth when women switch into high-amenity jobs and are more likely to stay there.

[15] Commute distance is also an important amenity that women value (Le Barbanchon, Rathelot and Roulet, 2020). However, it is beyond the scope of this paper because: (i) commute distance is not observed in the data; and (ii) it requires including both worker and job locations in the value functions which significantly complicates the model.

3 Model

Motivated by the data patterns of men and women's labor market behaviors, the model incorporates important gender differences to study both workers' and employers' decisions. I first describe the characteristics of workers and firms, and their life-cycle stages. I then explain the matching process between workers and firms and the wage determination mechanisms. Lastly, the steady-state equilibrium of the labor market is characterized.

3.1 The environment

Time is continuous and infinite. The labor market is populated by a continuum of female and male workers of measures ℓ and $(1-\ell)$, as well as a continuum of jobs of a fixed measure ι.

Workers are heterogeneous in their value for amenity ϵ, and accumulate human capital k through learning-by-doing. Upon entering the labor market, workers of gender $g \in \{m, f\}$ draw their initial skills and tastes for amenities from an exogenous discrete distribution with probability mass function $\xi_0^g(k, \epsilon)$. The model focuses only on workers' lives after graduation, and takes as given their pre-labor market decisions in human capital investment (including choices in the level of education and field of study).[16] Men's amenity preferences are assumed to be time-invariant, while women's preferences are allowed to change after childbirth.

A job is a 4-digit occupation within a firm. Jobs are heterogeneous in productivity p and amenity provision α drawn from an exogenous distribution with joint density $\varphi(p, \alpha)$. If a job is vacant, it does not produce any output and has to pay a flow vacancy cost c. Importantly, each job can match with only one worker, and employers are not allowed to search for new hires when the job is filled.[17] The distribution of jobs is fixed at $\varphi(p, \alpha)$ and there is no free entry of new vacancies.[18]

When a worker of type (k, ϵ) matches with a job of type (p, α), they produce $f(k, p)$

[16]To the extent that the field of study is highly correlated with occupation choice, the preference for different university majors is partially incorporated in the model as men and women are allowed to have different tastes for occupations.

[17]Assuming that the worker has an opportunity cost of accepting a job is standard in the literature, as search intensities might differ in employment and unemployment. It is thus also natural to assume that the employer incurs an opportunity cost of filling a job. While most studies assume that the marginal cost of hiring an additional worker for an already filled position is zero, my model posits that this cost is infinite. The reality might be somewhere in between where firms engage in some degree of replacement hiring (Kiyotaki and Lagos, 2007).

[18]The model can be easily extended to allow for a one-shot entry where employers can create new vacancies until the worst job makes zero profit. The important assumption is that the steady-state economy has a finite measure of jobs, and there is no vacancy creation after the initial entrance.

units of flow output (regardless of gender). Since the degree of sorting depends crucially on the supermodularity of the production function,[19] $f(.)$ is assumed to take a flexible CES form to allow for any degree of complementarity between worker and job productivities. In employment, workers receives a flow wage ω that corresponds to their bargained share of the match surplus. In non-employment, the worker's flow utility is assumed to take the form $b\,k$.[20]

The worker's human capital evolves stochastically according to a law of motion $\rho_e(k, p)$ in employment and $\rho_u(k)$ in unemployment. Human capital k is assumed to be general and transferable across different job types.[21] The skill accumulation rate in employment is allowed to depend on job productivity p, capturing the idea that workers might learn faster on the job when matched with more productive employers (Gregory, 2020), either from knowledge spillovers by more productive coworkers (Nix, 2019), or from doing more complex tasks (Caines, Hoffmann and Kambourov, 2017).

While production and human capital evolution are assumed to be symmetric between men and women, several transition rates are allowed to be different by gender especially in fertile ages. Workers go through four age segments in life. All workers start their careers in a stage with no child (the NC stage). At an exogenous fertility rate χ, the worker has a child and enters a stage with young child (the YC stage). Every time the worker has a child, he/she will enter a parental leave (PL) stage and stay out of the labor force. Men and women might stay in the PL stage for different durations governed by exit rates η_m and η_f, upon which they can go back to their previous employers. Workers can have children repeatedly until they become non-fecund (NF) at rate γ. They retire at rate ϕ in NF stage, and new workers enter the labor market at the same rate. Within each age segment $t \in$ {NC, PL, YC, NF} of life, the search and matching process is analogous.

To simplify notation, let $\boldsymbol{x} = (k, \epsilon)$ and $\boldsymbol{y} = (p, \alpha)$ denote worker and job heterogeneity respectively.

3.2 Gender and family

The model assumes that men and women make decisions independently, even when living in the same household. This is equivalent to solving the household's problem jointly

[19] Eeckhout and Kircher (2011) shows that the cross-partial of the production function determines the strength of sorting in realized matches, but the sign of sorting cannot be identified by wage data alone.

[20] The assumption that home productivity is proportional to the worker's skills k is common in the literature (for example in Cahuc, Postel-Vinay and Robin (2006); Flinn, Todd and Zhang (2024)).

[21] If some human capital were occupation-specific or firm-specific, that would likely amplify the role of employers in gender wage gaps. This is because firms' costly investments in job-specific human capital would yield lower returns if the worker has shorter or more interrupted employment spells.

since flow utility is assumed to be linear in both employment and non-employment.[22]

Fertility is exogenous in the baseline model. In Section 4.5, I estimate an alternative specification with endogenous fertility and show that it is not supported by the observed fertility patterns. Gender differences in parental leave duration (governed by η^g) and in separation rates (δ^g_{NC} and δ^g_{YC}) are also modeled as exogenous and do not respond to wages. The model allows for endogenous quits, which may differ by gender, in response to changes in career prospects.[23] Although parental leave decisions may in practice be made jointly within households, I assume that they are largely shaped by social norms and family leave policies rather than by wage offers. The primary objective of this paper is to study employers' responses to women's roles in the family, taking the household division of labor as given. I discuss future extensions to this modeling choice in Section 6.

Men and women's tastes for family-friendly amenities are drawn from different distributions, and women's preference is also allowed to change after having children. These preference parameters can represent genuine taste or liking for job amenities, but they may also stem from the couple's joint optimization, reflecting the constraints in their coordination (which are not modeled).

Workers make labor supply decisions on the extensive margin (whether to accept a job or not), but they do not explicitly choose how many hours to work.[24] However, the level of family-friendly amenities α is related to the overtime demands of the job, so switching to a job with higher levels of α might entail lower hours.

3.3 Search and matching

At each point in time, workers can be matched to a job, unemployed, or on parental leave. The aggregate number of meetings between vacancies and searching workers is determined by a standard aggregate matching function $m(\hat{U}, V)$. This takes as inputs the total number of vacancies V and the total amount of effective job seekers $\hat{U} = U + s(1 - U)$, where U is the total number of unemployed workers and s is the search intensity in employment relative to unemployment. The matching function is assumed to be increasing

[22] As noted in Dey and Flinn (2008) and Guler, Guvenen and Violante (2012), the linear utility assumption allows the household's maximization problem to be decentralized. Assuming non-linear utility will complicate the couple's joint-search problem substantially, since the spouse's values have to be taken into account when bargaining with employers.

[23] Endogenous quits alone cannot generate the sizable gender gap in quit rates observed in the data. Women's transitions from employment to unemployment may reflect factors beyond wages and career opportunities; therefore, exogenous gender differences in the δ's are necessary to match the transition rates in the data.

[24] See Burdett and Mortensen (1978) and Flabbi and Mabli (2018) for search models that incorporate women's participation and hours decisions.

in both arguments and exhibit constant returns to scale.

For ease of exposition, let $\lambda = \frac{m(\hat{U},V)}{\hat{U}\,V}$ summarize the effect of market tightness. λ is constant in a stationary equilibrium, but it is not invariant to policy, and it is important to allow it to change when evaluating interventions or counterfactual regulations.

Let $u_t^g(\boldsymbol{x})$ denote the measure of unemployed workers of gender g, age t and type $\boldsymbol{x}$, and let $v(\boldsymbol{y})$ denote the measure of vacancies of type $\boldsymbol{y}$. The joint distribution of matches between workers of type $\boldsymbol{x}$ and jobs of type $\boldsymbol{y}$ is denoted as $h_t^g(\boldsymbol{x},\boldsymbol{y})$. While unemployed, workers randomly sample offers from the vacancies distribution, and the instantaneous rate at which an unemployed worker meets a vacancy of type $\boldsymbol{y}$ is $\lambda\, v(\boldsymbol{y})$. Similarly, employed workers meet vacancies at rate $s\lambda\, v(\boldsymbol{y})$, and vacancies meet employed workers at rate $s\lambda\, h_t^g(\boldsymbol{x},\boldsymbol{y})$.

Upon a meeting between a worker and a job, a match will be formed if it generates positive surplus. In other words, match formation is assumed to be efficient.

Let $U_t^g(\boldsymbol{x})$ denote the lifetime value of an unemployed worker of type $\boldsymbol{x}$, $J(\boldsymbol{y})$ denote the vacancy value of a job of type $(\boldsymbol{y})$. Let $P_t^g(\boldsymbol{x},\boldsymbol{y})$ denote the value of joint production of a match between worker $\boldsymbol{x}$ and job $\boldsymbol{y}$. The surplus of a match is defined as $S_t^g(\boldsymbol{x},\boldsymbol{y}) = P_t^g(\boldsymbol{x},\boldsymbol{y}) - U_t^g(\boldsymbol{x}) - J(\boldsymbol{y})$. A match is feasible and sustainable if the match surplus is positive.

Workers have bargaining power denoted by β and obtain a share of the match rent. Let $W_t^g(\omega,\boldsymbol{x},\boldsymbol{y})$ and respectively $\Pi_t^g(\omega,\boldsymbol{x},\boldsymbol{y})$ denote the value of a wage contract ω for the worker and the value for the employer, respectively. The surplus can be written as:

$$S_t^g(\boldsymbol{x},\boldsymbol{y}) = \underbrace{W_t^g(\omega,\boldsymbol{x},\boldsymbol{y}) - U_t^g(\boldsymbol{x})}_{\text{Worker's share}} + \underbrace{\Pi_t^g(\omega,\boldsymbol{x},\boldsymbol{y}) - J(\boldsymbol{y})}_{\text{Employer's share}}.$$

The way in which wage ω splits the surplus between the worker and the employer will be discussed in the following section.

3.4 Wage determination

To define wages and renegotiations, I follow the setup in Dey and Flinn (2005) and Cahuc, Postel-Vinay and Robin (2006). Workers' wages are determined by sequential auctions. Different wages are negotiated when a worker leaves unemployment, and when counteroffers are made for an employed worker upon poaching events.

Wage bargaining with unemployed workers The starting wage $\omega_{0t}^g(\boldsymbol{x},\boldsymbol{y})$ obtained by a type-$\boldsymbol{x}$ unemployed worker when matched with a type-$\boldsymbol{y}$ job is such that the worker receives the reservation utility $U(\boldsymbol{x})$ plus a share β of the surplus:

$$W_{0t}^{g}(\boldsymbol{x},\boldsymbol{y}) = U_t^g(\boldsymbol{x}) + \beta\, S_t^g(\boldsymbol{x},\boldsymbol{y}) \tag{1}$$

for jobs where surplus $S_t^g(\boldsymbol{x},\boldsymbol{y})$ is positive.

Wage at job-to-job transitions When a worker of type $\boldsymbol{x}$ encounters an alternative job package $\boldsymbol{y}'$ that produces more surplus than her current job, she will switch jobs with a wage $\omega_{1t}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')$ such that the value she receives at the new job $\boldsymbol{y}'$ is $W_{1t}^g(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')$. In this scenario, the worker extracts the maximum value from the incumbent match $P_t^g(\boldsymbol{x},\boldsymbol{y}) - J(\boldsymbol{y})$ plus a β share of the surplus difference:

$$W_{1t}^{g}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') = P_t^g(\boldsymbol{x},\boldsymbol{y}) - J(\boldsymbol{y}) + \beta\left[S_t^g(\boldsymbol{x},\boldsymbol{y}') - S_t^g(\boldsymbol{x},\boldsymbol{y})\right] \tag{2}$$

Wage renegotiation upon poaching If the poaching job $\boldsymbol{y}'$ generates a match surplus below that of the incumbent job, i.e. when $S_t^g(\boldsymbol{x},\boldsymbol{y}') < S_t^g(\boldsymbol{x},\boldsymbol{y})$, the worker will stay in the same job. Incumbent employers will respond to outside offers and update wages only when there is a credible threat – when either the worker or the employer will credibly separate if they do not obtain an improved offer. In other words, wages will be re-negotiated when the poaching firm offers a value greater than what the worker currently receives. In this case, wages will be updated to $\omega_{2t}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})$ such that the worker receives an updated value $W_{2t}^g(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})$ at the incumbent job $\boldsymbol{y}$:

$$W_{2t}^{g}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) = P_t^g(\boldsymbol{x},\boldsymbol{y}') - J(\boldsymbol{y}') + \beta\left[S_t^g(\boldsymbol{x},\boldsymbol{y}) - S_t^g(\boldsymbol{x},\boldsymbol{y}')\right] \tag{3}$$

When a worker's human capital changes from k to k', her wage adjusts so that the continuation value reflects what a type k' worker would obtain under either the W_0 or W_2 contract with the same outside offer $\boldsymbol{y}'$, whichever is higher. Please refer to Appendix E for details of the workers' values.

3.5 Value functions

In order to define an equilibrium, I will describe the value functions and the distributions of workers and jobs across employment states and life stages. These define the decision rules for each agent.

3.5.1 Value in unemployment

In the *No Child* stage of life, the utility of an unemployed worker is:

$$\begin{aligned}\Big(r + H_u(k) + \chi + \gamma\Big)\, U_{NC}^g(\boldsymbol{x}) = b(k) + \chi\, U_{PL}^g(\boldsymbol{x}) + \gamma\, U_{NF}^g(\boldsymbol{x}) + \sum_{k'} \rho_u(k'|k)\, U_{NC}^g(k',\epsilon) \\ + \sum_{\boldsymbol{y}} \lambda\, v(\boldsymbol{y})\;\beta\left[S_{NC}^g(\boldsymbol{x},\boldsymbol{y})\right]^+ \end{aligned} \tag{4}$$

where r is the risk-free interest rate, and $[S]^+$ denotes the maximum operator $\max\{S, 0\}$. The worker's human capital level in unemployment evolves from k to k' at Poisson rate $\rho_u(k'|k)$, and the total rate of human capital change is denoted by $H_u(k) = \sum_{k'} \rho_u(k'|k)$.

The worker is subject to life-cycle shocks. When an unemployed worker has a child at rate χ, he/she exits the labor market and does not conduct job search in the *Parental Leave* stage. When parental leave terminates at rate η^g, the unemployed worker enters the labor market and resumes job search in unemployment in the *Young Child* stage. At any point in life, the worker ages at rate γ, upon which he/she enters a non-fecund period. The details of the unemployment values in PL, YC and NF stages are described in Appendix F.

3.5.2 Value of vacancy

A vacant job could potentially hire a male or female worker of any age $t \in \{NC, YC, NF\}$. The value of a vacancy of type $\boldsymbol{y}$ is:

$$\begin{aligned} r\,J(\boldsymbol{y}) = & -c + \sum_{t,g,\boldsymbol{x}} \lambda\, u_t^g(\boldsymbol{x})\,(1-\beta)\Big[S_t^g(\boldsymbol{x},\boldsymbol{y})\Big]^+ \\ & + \sum_{t,g,\boldsymbol{x},\boldsymbol{y}'} s\lambda\, h_t^g(\boldsymbol{x},\boldsymbol{y}')\,(1-\beta)\Big[S_t^g(\boldsymbol{x},\boldsymbol{y}) - S_t^g(\boldsymbol{x},\boldsymbol{y}')\Big]^+ \end{aligned} \tag{5}$$

where c is the flow cost of vacancy. Job vacancies have the opportunities to meet unemployed and employed workers of any age, gender, productivity and preference types. Since employers have capacity constraints, the option value of waiting J is typically positive.

3.5.3 Joint value of a match

In the *No Child* stage, the joint value of a match between worker $\boldsymbol{x}$ and job $\boldsymbol{y}$ is:

$$\begin{aligned} \Big(r + H_e(k,p) + \delta_{NC}^g + \chi + \gamma\Big)\, P_{NC}^g(\boldsymbol{x},\boldsymbol{y}) = & \underbrace{(1-\tau) f(k,p)}_{\text{after-tax flow output}} + \underbrace{q(\epsilon,\alpha)}_{\text{value for amenities}} + \underbrace{\sum_{\boldsymbol{x}'\neq\boldsymbol{x}} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\, \widetilde{P}_{NC}^g(\boldsymbol{x}',\boldsymbol{y})}_{\text{HC accumulation}} \\ & + \underbrace{\delta_{NC}^g\Big(J(\boldsymbol{y}) + U_{NC}^g(\boldsymbol{x})\Big)}_{\text{exogenous separation}} + \underbrace{\chi\, \tilde{P}_{PL}^g(\boldsymbol{x},\boldsymbol{y})}_{\text{fertility}} + \underbrace{\gamma\, \widetilde{P}_{NF}^g(\boldsymbol{x},\boldsymbol{y})}_{\text{ageing}} \\ & + \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\; \beta\Big[\underbrace{S_{NC}^g(\boldsymbol{x},\boldsymbol{y}')}_{\text{poaching job surplus}} - \underbrace{S_{NC}^g(\boldsymbol{x},\boldsymbol{y})}_{\text{current job surplus}}\Big]^+ \end{aligned} \tag{6}$$

The match between worker of human capital k and job of productivity p produces $f(k,p)$ units of flow output, regardless of gender. There is a proportional tax τ on the flow output to finance parental leave benefits. The worker enjoys a flow utility that is a function of his/her value for amenities ϵ and the level of amenity provision at the job α.

The worker's human capital in employment evolves from k to k' at Poisson rate $\rho_e(k'|k,p)$, where the total hazard is denoted by $H_e(k,p) = \sum_{k' \neq k} \rho_e(k'|k,p)$. Upon exogenous separation δ^g_{NC}, the match dissolves and the worker and the employer both receive their outside options. The worker searches on-the-job, and employers Bertrand-compete for the worker.

All matches are efficient, and an existing match is allowed to endogenously dissolve if the joint value of the match falls below the sum of the agents' outside options in separation. There could be endogenous quits when human capital level k changes or when life stage t changes:

$$\widetilde{P}^g_t(\boldsymbol{x},\boldsymbol{y}) = \max\left\{P^g_t(\boldsymbol{x},\boldsymbol{y}),\ J(\boldsymbol{y}) + U^g_t(\boldsymbol{x})\right\}, \qquad t = \{NC, PL, YC, NF\}$$

3.5.4 Parental leave

When a worker has a child, several changes take place. The woman's utility from amenities changes from $q(\epsilon,\alpha)$ to $q^f_{YC}(\epsilon,\alpha)$, whereas the men's value stays the same. Exogenous separation rates also change from δ^g_{NC} to δ^g_{YC}. There is no human capital accumulation and no job search in the *Parental Leave* stage. The joint value in parental leave is:

$$(r + \delta^g_{YC} + \eta^g + \gamma)\, P^g_{PL}(\boldsymbol{x},\boldsymbol{y}) = (1-\tau) \underbrace{R\, f(k,p)}_{\text{reduced flow output}} + \underbrace{q^g_{YC}(\epsilon,\alpha)}_{\text{value for amenities}} + \underbrace{\delta^g_{YC}\Big(J(\boldsymbol{y}) + U^g_{PL}(\boldsymbol{x})\Big)}_{\text{exogenous separations}}$$
$$+ \underbrace{\eta^g\, \widetilde{P}^g_{YC}(\boldsymbol{x},\boldsymbol{y})}_{\text{PL ends}} + \underbrace{\gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y})}_{\text{ageing}} \tag{7}$$

A detailed description of the Finnish parental leave system is provided in Appendix D. Mimicking this institutional setting, the model incorporates three key features. First, both men and women take parental leave after childbirth, although average leave durations may differ substantially by gender. Second, jobs are protected during leave, so employers must keep positions available upon the worker's return. Third, matches continue to produce output while the worker is absent, but at a reduced level equal to a fraction R of pre-childbirth productivity.

The parameter R captures, in reduced form, the adjustment costs firms face when workers take parental leave. Even though Finnish employers do not directly finance leave benefits, they may incur costs from hiring temporary replacements or reallocating tasks among existing workers, often resulting in lower productivity. Evidence from Sweden suggests that such adjustments can be substantial (Ginja, Karimi and Xiao, 2023).[25]

From a modeling perspective, the employer continues to share the match surplus with the worker on leave, maintaining a tractable transferable utility (TU) framework in which joint match values can be analyzed without separately tracking worker and firm payoffs.[26] This abstraction provides a parsimonious way to incorporate employers' adjustment costs while keeping the model analytically and computationally manageable.

Workers on leave are by default associated with their previous employers (in both the data and model), but can separate from their employers exogenously or endogenously in the parental leave period. Women and men finish parental leave at rate η^f and η^m respectively, upon which unemployed workers start searching for jobs and employed workers go back to pre-birth employers. The worker can have another child any time during fertile ages (including during parental leave). Upon having another child while employed, the worker will go into parental leave again.

The transition parameters and preference parameters in *Young Child* stage are the same as in *Parental Leave* stage, and one should think of these two stages as the period where workers have young children at home. The only difference is that individuals in *Parental Leave* stage are matched with some employers but are not working, whereas those in YC stage are actively participating in the labor force.

In stage NF, individuals are non-fecund and will not have any additional child. In this period, men and women have the same separation rate δ, and retire at rate ϕ. The details of the match values in YC and NF stages are described in Appendix F.

3.6 Steady-state balance flow conditions

In equilibrium, all agents follow their optimal strategy. Denote the measure of workers of gender g in age segment $t \in \{NC, PL, YC, NF\}$ as m_t^g. The total measure of women of all ages should add up to $\ell^f = \ell$, and men to $\ell^m = 1 - \ell$.

$$m_{NC}^g + m_{YC}^g + m_{PL}^g + m_{NF}^g = \ell^g \tag{8}$$

Also, the flows into and out of each age segment should balance.

[25]Ginja, Karimi and Xiao (2023) finds that firms hired temporary workers and increased incumbents' hours when parental leave was extended by 3 months. The associated reorganization costs were equivalent to about 10 full time months of wages per additional worker on leave.

[26]In practice, workers on leave receive government funded leave benefits that may differ from $\beta \cdot Rf(k,p)$, the worker's share of the reduced output. Modeling this explicitly would require relaxing the transferable utility assumption, which would significantly complicate the model. In the estimation, $R = 0.7$ based on Ginja, Karimi and Xiao (2023), while the average replacement rate in the data is about 63 percent, so the two are reasonably close.

$$\chi\,(m^g_{NC} + m^g_{YC}) = (\gamma + \eta^g)\,m^g_{PL} \tag{9}$$
$$\eta^g\,m^g_{PL} = (\chi + \gamma)\,m^g_{YC} \tag{10}$$
$$\gamma\,(m^g_{NC} + m^g_{YC} + m^g_{PL}) = \phi\,m^g_{NF} \tag{11}$$

The equilibrium distribution of vacancies and matches will satisfy the following accounting equation:

$$v(\boldsymbol{y}) + \sum_{g,\boldsymbol{x},t} h^g_t(\boldsymbol{x},\boldsymbol{y}) = \varphi(\boldsymbol{y}), \quad t \in \{NC, YC, PL, NF\} \tag{12}$$

where the total measure of all jobs is fixed at $\sum_{\boldsymbol{y}} \varphi(\boldsymbol{y}) = \iota$.

The equilibrium distribution of workers is such that the flows into and out of any worker stock must balance for each worker type $(g, \boldsymbol{x}, t)$, in employed or unemployed state, across all job types (if employed). Appendix G describes the steady-state balanced flow conditions.

3.7 Definition of equilibrium

A stationary equilibrium is a tuple of value functions $\{U^m, U^f, P^m, P^f, J\}$ together with a distribution of male and female workers across employment states and across job types $\{u^m, u^f, h^m, h^f\}$ as well as a distribution of job vacancies v such that:

(i) The value functions satisfy Bellman Equations (4) to (7) and those in Appendix F.

(ii) The distributions $\{u^m, u^f, h^m, h^f, v\}$ are stationary given the transitions implied by the value functions, and satisfy balanced flow conditions (8) to (12) and flow equations in Appendix G.

(iii) Equilibrium wages are determined by surplus sharing rules defined in (1) to (3).

Note that the equilibrium values and allocations (points (i) and (ii) above) can be solved without making any reference to wages, just like in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). This is because utility is transferable between the worker and employer, so joint values and surpluses do not depend on wages. Moreover, match formation and worker mobility decisions are determined only by the surpluses or difference in surpluses between two jobs, so the equilibrium worker and job allocations also do not depend on wages. The advantage of this transferable utility framework is that it makes the model very tractable, and the computation of the equilibrium fairly feasible.

There is no analytical solution to the equilibrium. Given the richness in the model structure designed to reflect characteristics of the Finnish labor market, proving the existence and uniqueness of the equilibrium is virtually impossible. I solve for the equilibrium numerically. To ensure convergence, I introduce a small i.i.d. logistic shock upon each job arrival that smooths workers' match formation decisions when two jobs offer nearly identical surplus values; the variance of this shock is fixed at a small value so it does not affect the quantitative results. Details are provided in Appendix H. I show that the solution always converges to the same equilibrium values and distributions when starting with different initial guesses of the equilibrium objects.

4 Estimation

In this section, I estimate the model using Simulated Method of Moments (SMM).[27] To this aim, I obtain a vector of moments from N individuals in the data, $\hat{m}^D = \frac{1}{N}\sum_{i=1}^{N} m_i$, for example mean wages out of unemployment in the first five years after graduation, etc. Model counterparts to these moments, $\hat{m}^S(\theta) = \frac{1}{M}\sum_{j=1}^{M} m_j^D$, are obtained from M simulated lives from the model based on a parameter vector θ. The estimation involves finding the vector θ that brings the simulated moments as close as possible to the data moments, i.e. minimizing the criterion function

$$L(\theta) = (\hat{m}^D - \hat{m}^S(\theta))^T \ \hat{W}^{-1} \ (\hat{m}^D - \hat{m}^S(\theta))$$

where $\hat{W}$ is a weighting matrix. Key parameters of interest are outlined below.

4.1 Model specification

The worker's human capital can take one of K discrete values $k \in \{k_1, k_2, ..., k_K\}$ and $0 < k_1 < k_2 < ... < k_K$. Human capital accumulation is assumed to take the form

$$\rho_e(k_{i+1} \,|\, k_i, p) = d_1 + d_2\, p, \qquad i \in \{1, 2, ..., K-1\}.$$

That is, an employed worker moves up by one category of human capital at a Poisson rate that is linear in his/her job productivity p. This captures the idea that workers might learn faster on the job when matched with more productive employers. The worker's human capital is assumed to stay unchanged while he/she is not working.[28]

When non-employed, the worker receives a flow utility of home production that takes a proportional form $b\,k$. While employed, the flow output produced by the match is specified by a flexible CES production function:

$$f(k, p) = A\left[a\, k^{\sigma} + (1-a)\, p^{\sigma}\right]^{\frac{1}{\sigma}}.$$

Central to the model is the matching between workers and jobs, which is intimately

[27]See for example McFadden (1989) and Pakes and Pollard (1989). Constructing the likelihood function for this model is intractable.

[28]The estimation can easily allow human capital to depreciate when non-employed, though there is no reason to think it would substantially change the counterfactuals.

related to the production function. A CES function allows for various degrees of complementarity between the worker's human capital and the employer's productivity, governed by the estimated value of σ. If $\sigma < 1$, the production function is supermodular, whereas if $\sigma > 1$, it is submodular. The CES function is a generalization of the commonly used multiplicative form $k\,p$ (for example in Cahuc, Postel-Vinay and Robin (2006) and Flinn, Todd and Zhang (2024)).

Men and women draw their values for amenities ϵ^m and ϵ^f from normal distributions $N(\mu_m, 1)$ and $N(\mu^f, 1)$ respectively.[29] In the *No Child* stage, the flow utility from amenities takes the simple form $q(\epsilon^g, \alpha) = \epsilon^g\,\alpha$. Women's value increases by M in motherhood, so that $q^f_{YC} = (\epsilon^f + M)\,\alpha$ in YC and PL stages, whereas men's values stay the same $q^m_{YC} = \epsilon^m\,\alpha$.

Finally, I assume the matching function has an elasticity of 0.5 and takes the functional form (see Petrongolo and Pissarides (2001)):

$$m(\hat{U}, V) = \vartheta\sqrt{\hat{U}\,V}$$

where effective job seekers $\hat{U} = U_{NC} + s_U(U_{YC} + U_{NF}) + s_E(1 - U_{NC} - U_{YC} - U_{NF})$. I allow search in unemployment to be different in early and late stages in life. The search intensity for the unemployed in NC stage is normalized to one, and that of the unemployed in YC and NF stages will be s_U. The relative search intensity of the employed is s_E and does not vary over the life-cycle.

The initial human capital of male and female workers follow exponential distributions $\xi^g_0(k) \sim \text{Exponential}(\xi_g)$. The initial human capital distribution is assumed to be the same within each worker preference type.

4.2 Job types

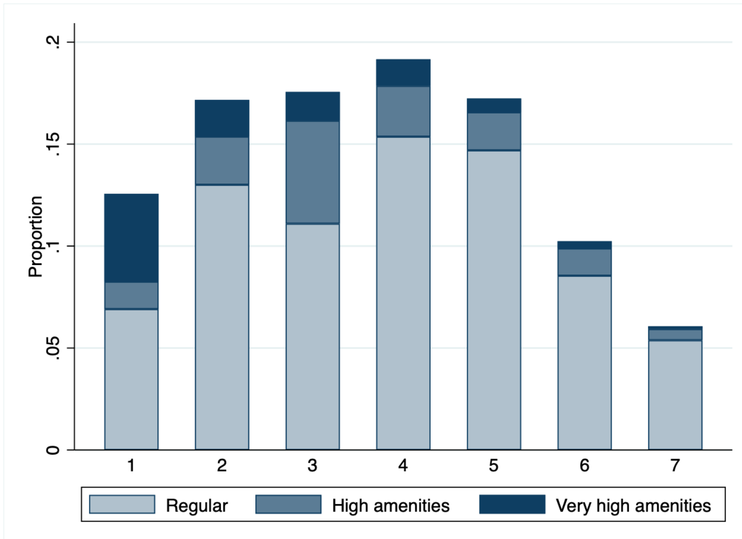
Job productivity types are discrete and not directly observed from the data. Since sorting and wages are endogenous to worker and job types in this model, the ideal estimation method would involve updating job classifications in iterative steps using an Expectation-Maximization (EM) algorithm such as that in Lentz, Piyapromdee and Robin (2023). However, the model in this paper does not yield analytical moments or closed-form expressions for the likelihood function. The estimation process must rely on simulation-based methods, which makes the iterative procedure computationally infeasible.

Therefore, I follow Bonhomme, Lamadon and Manresa (2019) (henceforth BLM) and

[29] The standard deviations of the distributions are normalized to 1 because of identification issues discussed in Taber and Vejlin (2020): in a standard search model with worker bargaining and preference for job amenities, the bargaining power parameter cannot be separately identified from the scale of utilities.

pre-classify job types in one step through k-means clustering. BLM shows that firm classes are identified from within-firm wage CDFs when there are sufficiently many workers per firm. However, that is not the case in my setting because many jobs (firm-occupation cells) have less than 5 workers.[30] Instead of wage distributions, I use the long-term average wage (between 1995 and 2013) in each firm-occupation cell to classify job types.

FIGURE 5. Distribution of jobs by productivity and amenities



NOTES: This figure shows the measure of jobs across productivity and amenity categories. Jobs are classified into seven productivity types using k-means clustering, where 1 denotes the lowest and 7 the highest productivity. Amenity provision is divided into three categories based on the job amenity index constructed in Appendix C: very high (more than 1 s.d. above the mean), high (between 0.5 and 1 s.d. above the mean), and regular (the remaining jobs).

Even though job classes cannot be identified from average wages alone under NAM (negative assortative matching), they are identified under PAM (positive assortative matching) because long-run average wages are monotonic in job productivity in this case.[31] Assuming PAM ex-ante for the estimation is less ideal. However, researchers have found overwhelming evidence of positive sorting between workers and firms in many countries.[32] Moreover, an advantage of using average wages to classify jobs is that there is a clear hierarchy of job types, which is useful when analyzing career progression over the

[30]The research question calls for a granular definition of jobs, since career progression of men and women might involve both moving up the occupational ranks within a firm as well as advancing toward better firms.

[31]Intuitively, highly-productive workers sort into highly-productive jobs under PAM, so average wages are increasing in job productivity type in steady-state equilibrium. This is shown in BLM's Supplemental Materials for a variation of the model of Shimer and Smith (2000) which is very similar to mine.

[32]For example in the US (Lopes de Melo, 2018; Bonhomme, Holzheu, Lamadon, Manresa, Mogstad and Setzler, 2023), Sweden (Bonhomme, Lamadon and Manresa, 2019), Denmark (Bagger and Lentz, 2018), Germany (Hagedorn, Law and Manovskii, 2017), Italy (Bartolucci, Devicienti and Monzón, 2018) amongst others.

life-cycle.

I estimate seven productivity clusters, where the support of the distribution is normalized so that the bottom type takes a productivity value of 1. Workers' human capital levels have the same support. Table I1 provides summary statistics about the job productivity categories. Each job is also assigned an amenity type based on its amenity index (constructed in Appendix C). The empirical distribution of jobs across both productivity and amenity dimensions is shown in Figure 5.

4.3 Estimation method and identification

Given the above specification, I simulate the model in continuous time and estimate two sets of parameters in an iterative procedure. The first set of parameters $\mathbf{\Lambda} = (\boldsymbol{\delta_t^g}, \vartheta, s_U, s_E)$ includes a vector of exogenous separation rates $\boldsymbol{\delta}_t^g$ and parameters from the matching function. The second group includes "core" model parameters characterizing human capital processes, production functions, bargaining, preferences, and initial distributions, denoted by $\mathbf{\Theta} = (d_1, d_2, A, a, \sigma, \beta, b, \mu_m, \mu^f, M, \xi^m, \xi^f)$.

Note that separation rates, job-finding rates and job-to-job transition probabilities in the model depend on equilibrium surplus values and the equilibrium distribution of vacancies, and consequently cannot be obtained independently outside of the model. However, given the equilibrium surpluses, parameters in $\mathbf{\Lambda}$ are directly related to workers' transitions in and out of work and between jobs. Therefore, $\mathbf{\Lambda}$ can be identified given $\mathbf{\Theta}$. Following Meghir, Narita and Robin (2015), I estimate the two groups of parameters using an iterative procedure that significantly reduces estimation time. Appendix I provides details on the estimation procedure and computation of standard errors.

Several points are worth noting regarding identification. The main assumptions and arguments are as follows.

Discrimination Based on fertility-related turnover concerns, employers' gender-specific wage-setting and job allocation decisions depend on both $\mathbf{\Lambda}$ and $\mathbf{\Theta}$ parameters in the model. These decisions can be distinguished from taste-based discrimination with the following intuition. After people become non-fecund (at age 45 on average), men and women face very similar model primitives as they no longer have children. Therefore, any gender wage gap after age 45, after conditioning on human capital histories up to that point, cannot be attributed to statistical discrimination based on child-related concerns. Any taste-based discrimination will be captured in the model residual, and cannot be separated from initial productivity differences between men and women (ξ_0^m and ξ_0^f).

Job types Once job clusters are estimated, they are taken as observed. This significantly simplifies the identification problem, as mobility patterns across high- and low-amenity jobs (within the same productivity category) help to identify workers' preferences for amenities, and mobility patterns across high- vs. low-productivity jobs (within the same amenity category) reveal the extent of production complementarity.

Preferences Amenity preference parameters μ^m and μ^f govern workers' mobility patterns across jobs of high- and low- amenity types, and do not affect the production of output. Therefore, the proportion of female workers in high-amenity jobs helps pin down the magnitude of μ^f relative to μ^m. The increase in value for amenities during motherhood M is closely linked to the proportion of women who switch into high-amenity jobs immediately after childbirth.

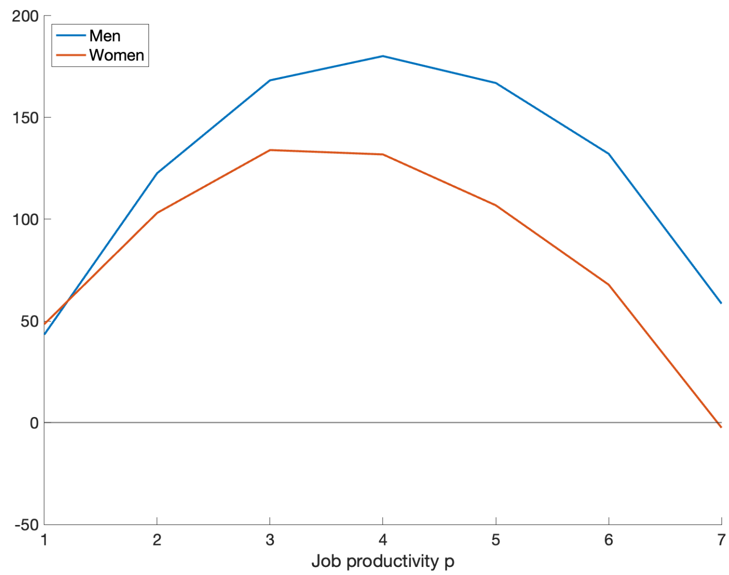
Amenity preferences also affect wages through compensating differentials. The wage differences between high- and low- amenity jobs of the same productivity type can help to identify μ^f and μ^m. However, it is important to note that worker's human capital (HC) is not observed in the data, so the wage differences include two opposing effects: first, compensating differentials push wages down in high-amenity jobs; and second, high-amenity jobs are more able to attract high-HC workers, making the average wage higher since the workers there are more positively selected. Both effects are in the model, and corresponding data moments would speak to which force dominates.

Human capital Human capital growth parameters d_1 and d_2 do not have a direct data counterpart since the assignment of workers to jobs is not random. However, with the aid of the full equilibrium structure of the model, these parameters can be related to the following aspects of the data. When a worker goes through an unemployment spell in the model, she falls off the job ladder and loses any "search capital" accumulated through job-to-job transitions. However, human capital is general and she will carry her accumulated experience to the next job. Comparing the wages immediately following a transition from unemployment to employment (UE wages) at different points of the life-cycle can inform us of the average human capital growth rate d_1 in the economy.

Moreover, human capital growth in each productivity category p is related to within-job wage growth in jobs of high- versus low-productivity types. Although wage gains within a job also depend on renegotiations triggered by poaching firms, the amount of contact with poachers is disciplined by s_E and ϑ that are pinned down in the previous step. Therefore, the remaining within-job wage growth could be attributed to human capital growth.

Production function Key to identification of production function parameters is the sorting of men and women across jobs. When production is very complementary (σ very small or negative), the marginal return of employing a high-type worker is considerably higher for high-productivity jobs. In the presence of capacity constraints on the employer side, this implies that the match surplus might not be monotonically increasing in job productivity (Eeckhout and Kircher, 2011), since high-productivity jobs have a much higher option value of waiting for a better match.

FIGURE 6. An example of surplus values of medium-skilled workers in NC stage



NOTES: The solid lines plot the surplus values of a male and female worker in *No Child* stage across jobs of different productivity levels. The man and woman have the same amenity preference and same productivity (both of skill type k_3). The production function in this example assumes complementarity between worker and job productivities, with $\sigma = -0.9$.

Indeed, the values of match surplus might be an inverted-U shape, or even decreasing in job productivity for a low-type worker. The example in Figure 6 shows that with production complementarity, a medium-skilled worker is best matched with middle-level jobs where the surpluses peak. Top jobs (category 7, mainly managers) generate relatively low surpluses with mediocre workers, and this is more severe for women as they have higher turnovers and generate less surplus in general. High vacancy values of the top jobs imply that these employers might turn off matches with women even though they might still match with equally skilled men.

Consider the contrary case where production is perfectly substitutable ($\sigma = 1$), then there are no productivity gains from sorting compared to random matching. Surpluses will be monotonically increasing in job productivity for any given worker type. Since match values are typically lower for women than men, it would imply that low-productivity jobs are the ones that stop matching with women, and we would see different sorting patterns of men and women vis à vis the case where production is complementary.

Labor share Relative productivity of labor (parameter a) is closely related to human capital parameters and wage growth over the life-cycle. When human capital appreciates, production grows more when a is high. Although both d_1, d_2 and a are positively related to wage growth moments, they could have opposite implications for UE wage levels. The intuition is that when a increases, all jobs are much better off matching with high-HC workers when production is complementary, and top jobs are actually worse off matching with low-type workers given the increased option value of hiring high-types. In contrast, an increase in d_1 or d_2 invariably raises surpluses and UE wages of all matches. As a result, in early career stages when most workers do not have much human capital, we will see lower UE wages when a increases but higher UE wages when d_1, d_2 increase. The extent of these effects depends on the strength of complementarity.

Calibrated parameters I calibrate the life-cycle Poisson parameters (see Table I2). Fertility rate χ is calibrated to match the average number of children workers have (both men and women have 1.7 children on average). Ageing rate γ is set to match the number of years between graduation and age 45, and retirement rate ϕ is set so that individuals retire at age 60 on average. The rates at which parental leave ends for men and women, η_m and η_f, are calibrated to match the average length of parental leave taken for each child by men and women respectively.

Other calibrated parameters include R and c. The reduction in flow production R during parental leave is calibrated to the adjustment costs of extended parental leave estimated in Ginja, Karimi and Xiao (2023). The vacancy cost c is calibrated to that in Lise, Meghir and Robin (2016). The monthly risk-free interest rate r is set to 0.012.

4.4 Results

The parameter estimates and standard errors are presented in Table 1. The last column of Table 1 reports the sensitivity analysis (Andrews, Gentzkow and Shapiro, 2017) and shows the three most important moments for each parameter in estimation. The sensitivity analysis is in line with the identification arguments.

The human capital estimates show that workers' skill accumulation rate is positively related to job productivity – workers' human capital upgrades much faster when they work at highly productive firms. The estimates imply that in the bottom job productivity category, human capital appreciates at the rate of 0.011, whereas at the top of the distribu-

TABLE 1. Parameter Estimates

Parameters		Estimates	S.E.	Top 3 Sensitivity Moments
(A) Θ parameters				
Complementarity	σ	-14.491	1.167	M9, M4, M1
Relative productivity	a	0.848	0.021	M2, M6, M5
TFP	A	29.526	1.392	M1, M4, M3
Baseline HC rate	d_1	0.001	0.002	M2, M6, M5
Proportional HC rate	d_2	0.010	0.003	M5, M6, M2
Men's value for amenities	μ^m	0.767	0.084	M10, M7, M12
Women's value for amenities	μ^f	0.883	0.092	M10, M7, M12
Preference increase in motherhood	M	1.815	1.923	M12, M10, M11
Worker's bargaining	β	0.519	0.016	M2, M4, M5
Home productivity	b	5.109	0.897	M1, M11, M2
Initial HC distribution - men	ξ^m	2.201	0.936	M8, M3, M1
Initial HC distribution - women	ξ^f	2.798	1.013	M8, M3, M1
(B) Λ parameters				
Women's separation rate in NC	δ_{NC}	0.011	0.001	M15, M11, M12
Women's separation rate in YC	δ_{YC}	0.015	0.003	M15, M11, M9
Men's separation rate	δ	0.008	0.002	M15, M11, M4
Matching efficiency	ϑ	0.110	0.007	M14, M16, M11
Relative search intensity in unemployment	s_U	0.687	0.405	M14, M11, M9
Relative search intensity in employment	s_E	0.526	0.214	M16, M9, M10

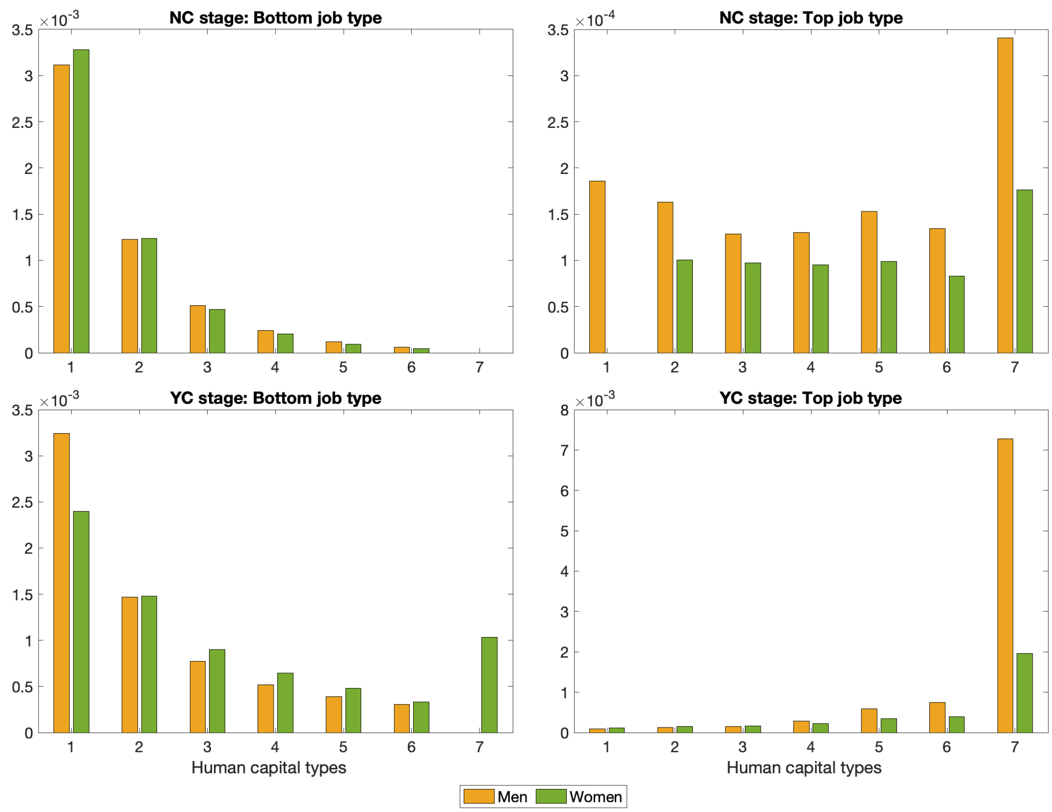
NOTES: Column "Top 3 Sensitivity Moments" reports the three most important moments for each parameter in estimation based on the sensitivity measure. M1, ..., M16 denote the 16 groups of targeted moments:

M1: Mean wages by gender (every 3 years)
M2: UE wages by gender (every 5 years)
M3: Initial gender wage gap (in year 1)
M4: Standard deviation (SD) of wages by gender (every 5 years)
M5: Within-p wage growth by gender from years 1-5 to years 21-25
M6: Within-p UE wage growth by gender from years 1-5 to years 21-25
M7: Wage difference between high vs. low amenity jobs (every 5 years)
M8: Initial distribution by gender (in year 1)
M9: Distribution of men and women across job productivity p (every 5 years)
M10: Proportion in high-α jobs by gender (every 5 years)
M11: Employment rate around childbirth by gender (every 5 years)
M12: Gender difference in high-α proportions around birth (every 5 years)
M13: Gender wage gap around birth (every 5 years)
M14: UE transition rates by gender in each life-cycle stage
M15: EU transition rates by gender around birth (every 5 years)
M16: Mean EE transition rates by gender

tion the rate is 0.034 (since p ranges from 1 to 3.28). There is a divergence in human capital levels of men and women over time, not only because men spend more time working and accumulating skills, but also because men are more represented at top jobs that offer better learning opportunities.

The estimate of σ implies an elasticity of substitution of 0.065, reflecting a high degree of complementary between worker and firm productivity.[33] Given the substantial gains from sorting, there are strong incentives for high-productivity workers to seek out high-productivity jobs, while low-HC workers gravitate toward lower-end jobs. Moreover, assortative matching becomes more pronounced over the life-cycle as mismatched workers gradually find matched jobs through on-the-job search.

FIGURE 7. Sorting of men and women in bottom vs. top jobs



NOTES: The bars represent the equilibrium sorting of men and women across jobs as implied by the parameter estimates. Each bar is the total measure of workers of a given HC type in a bottom job (productivity category 1) or in a top job (productivity 7). Both bottom and top jobs are of regular amenity provision type.

The baseline estimates imply an equilibrium allocation where the most productive

[33]Although using different models and data samples, many studies that estimate CES production function also find the complementarity parameter to be negative, ranging from -0.895 in Lise, Meghir and Robin (2016) to -7.3 in Bagger and Lentz (2018). My σ estimate of -14.49 is larger in magnitude, likely because I focus on a sample of highly educated workers, whose production function may exhibit stronger complementarity compared to the general population.

jobs (category 7) are less likely to match with women in the *No Child* stage. As shown in Figure 7, top-productivity jobs do not match with women of the lowest HC type, whereas the same jobs do match with low-HC men.

In other words, even before women have children, employers already apply a higher quality threshold when hiring women into top positions relative to men as they anticipate large gender differences after childbirth.[34] This matching pattern holds regardless of workers' amenity preferences. Such differential job allocations against women in early career could have long-term consequences given the different rates of skill accumulation across high- and low-productivity jobs.

In the *Young Child* stage, men of the highest HC type do not match with bottom jobs, whereas high-HC women are willing to take the low-productivity jobs in YC stage (see bottom left panel of Figure 7). This is because high-HC men have greater outside options and a high likelihood of matching with top jobs, so they would rather wait in unemployment than take a low-end job. In contrast, high-HC women have a lower reservation value than their male counterparts because women are subject to high separation rates in YC, so there is less value in waiting for better jobs to arrive.

In the *Non-Fecund* stage where workers have moved beyond child-rearing ages, match formation decisions are similar for men and women.

The preference estimates suggest that men and women have similar valuations for amenities before having children, but women's preference increases to almost twice as much after childbirth. However, women's movement into high-amenity jobs is not as pronounced and sudden in the model as in the data. This is because in a frictional environment in the model, opportunities to move to high-amenity jobs may not arise immediately after childbirth. Anticipating the rate of encountering high-amenity jobs, some women already sort into these jobs before childbirth and others gradually move into them after having children.

The model fits the life-cycle wage profiles of men and women very well, and is able to replicate key moments of the data. Appendix Figure I1 summarizes the fit of the model moments compared to targeted data moments. Men have higher wages than women throughout the life-cycle, are less represented in low-productivity jobs and more represented in high-productivity jobs. The proportion of women in high-amenity jobs increases after childbirth, and the gender wage gap increases in the first years after birth before

[34]This mirrors the finding in Bronson and Thoursie (2021), who show that the gender penalty in within-firm promotion probability prior to first birth accounts for about 30% of the cumulative promotion gap by age 45, consistent with statistical discrimination.

coming down 10 years afterwards. All these important qualitative features of the data are captured by the model.

4.5 Alternative models

I estimate two alternative model specifications to assess the robustness of the key modeling assumptions: one with no capacity constraint, to evaluate the role of equilibrium sorting; and one with endogenous fertility, to assess the appropriateness of the exogenous fertility assumption.

No capacity constraint

I estimate an alternative model without capacity constraints (akin to a conventional search model), and examine whether it can capture the main patterns in the data.

With no capacity constraints, free entry drives profits to zero and vacancy values are set to zero for all jobs. In this environment, sorting along the productivity dimension does not occur because surplus values increase monotonically with job productivity for all worker skill types. Since all workers rank jobs by productivity in the same way, matching with top jobs is essentially random. Sorting along the amenities dimension may still occur since men and women value amenities differently.

The parameter estimates are reported in Appendix Table J1. While this alternative model is able to match the wage profiles, it fails to replicate the sorting patterns of men and women over the life-cycle (Appendix Figure J1, panels (B) and (C)). Since higher levels of human capital do not make experienced workers more likely to move from low- to high-productivity jobs, the shares of men and women in high-productivity positions remain flat over the life-cycle, in contrast to the pronounced shifts observed in the data.

Importantly, because the surplus functions are no longer non-monotonic, top managerial positions match with both men and women of all types in this model, leading to nearly equal representation of men and women in top jobs.[35] As a result, a model without capacity constraints cannot account for the "glass ceiling" for women in top positions.

Endogenous fertility

To evaluate the baseline assumption of exogenous fertility, I estimate a variant of the model in which workers endogenously decide whether to have a child. The model is

[35] The parameter estimates imply slightly more men than women in top jobs, driven entirely by women's stronger preference for amenities and the concentration of high-amenity jobs among low-productivity positions. Since the estimated gender difference in amenity preferences is small, the model implies a gender gap in top positions that is far smaller than in the data.

described in detail in Appendix K.

In this specification, a worker in the *No Child* stage chooses to have a child only if the continuation value of having a child, $\tilde{P}_{PL}$, exceeds the value of remaining in *No Child* stage, P_{NC}. Flow utility is augmented by a "value for children" parameter κ in the *PL* and *YC* stages. Relative to the baseline, this specification estimates three additional parameters: the gender-specific fertility arrival rates χ^g and the flow utility for children κ.

I estimate the endogenous fertility model using the same targeted data moments as the baseline, supplemented by additional fertility moments: men and women's completed fertility by initial job category. The parameter estimates are presented in Appendix Table K1.

Despite matching aggregate fertility, the model generates a gradient across job types that does not align with the data. As shown in Appendix Figure K1 Panel (L), the endogenous fertility specification predicts a sharp decline in childbearing among women who begin their careers in more productive jobs. Since human capital accumulates rapidly in these jobs, the utility gain from children is insufficient to offset the forgone human capital associated with long parental leave spells, making these women more likely to stay in the *No Child* stage. Men's fertility behavior remains the same as in the baseline model under these estimates, since their short parental leave durations do not discourage childrearing.

These predictions are inconsistent with empirical evidence. In the data, both men and women with stronger initial jobs tend to have slightly *more* children over the life-cycle, not fewer.[36] By contrast, the baseline model with exogenous fertility aligns better with the observed fertility patterns, in which completed fertility varies little across job categories. Note that the endogenous fertility model does not nest the baseline. To compare the two, I evaluate the baseline model using the same set of moments as in the estimation of the endogenous fertility model, including the fertility moments. The resulting criterion function is 11% lower than that of the endogenous fertility model, indicating that the baseline provides a better fit despite having fewer parameters.

In summary, capacity constraints are necessary for the model to generate a "glass ceiling" effect in women's career progression, and the exogenous fertility assumption produces fertility patterns that more closely match the data. Therefore, the baseline model is preferred and is used for the counterfactual exercises that follow.

[36]One possible explanation is that childrearing is costly, so higher early-career earnings might encourage fertility. Incorporating such income effects of childbearing, however, lies beyond the scope of the model.

5 Gender gap decomposition and policy counterfactuals

Given the baseline model estimates, I first decompose the life-cycle gender gaps by sequentially shutting off additional channels. Then I compare three policies aimed at reducing gender inequality: (1) more parental leave months earmarked for fathers; (2) equal hiring at top jobs; and (3) equal pay for equal work.

5.1 Decomposition of the life-cycle gender wage gap

I first decompose the gender gaps in hourly log wages over the life-cycle. There is no straightforward way of decomposing the gender wage gap, since the channels interact with each other and the sequence of the exercises might matter. In the following decomposition, I focus on the impact of child-related career interruptions on human capital accumulation and its interactions with employers' decisions, while considering preference for amenities separately. Figure 8 shows the cumulative effects on the gender wage gaps when additional channels are implemented (see Table L1 for corresponding numbers). Table L2 conducts a different counterfactual exercise that shows the individual effect of one channel at a time (while holding other channels fixed), and illustrate any interactions between the mechanisms.

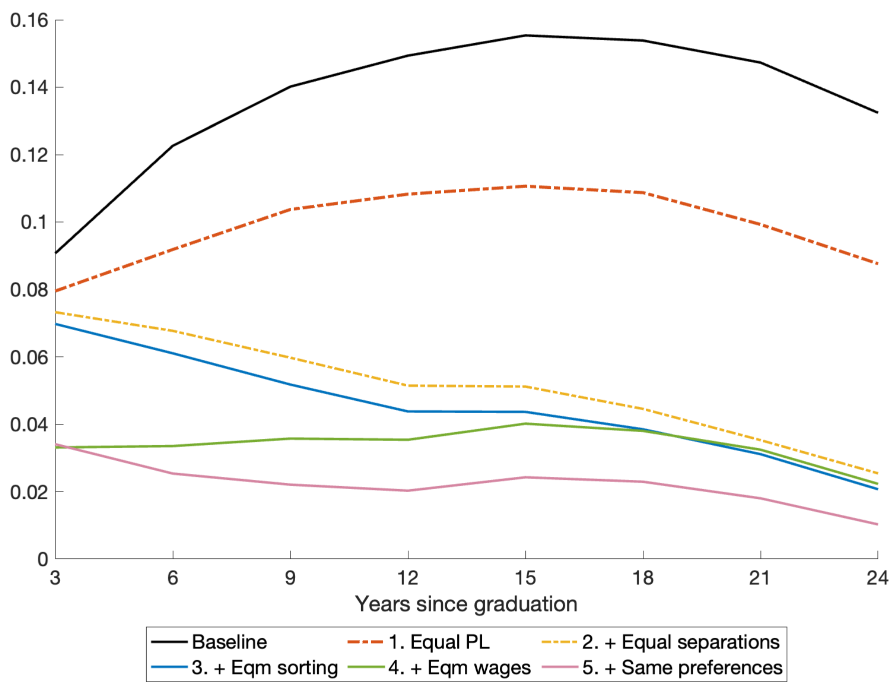
The top black solid line in Figure 8 shows the gender wage gap implied by model estimates. The model is able to replicate both the level and the hump-shaped pattern of the log wage gap in the data. Average wage gaps in three-year windows increase from 9.1 log points in the first three years to 15.5 log points in mid-career, and then declines to 13.2 log points later in the life-cycle (Table L1). The narrowing of the gender wage gap fifteen years after labor market entry is mainly driven by men already reaching a high point on the job ladder so they have little room for advancement, while women are returning from child-related career break and are starting to catch up.

The life-cycle gender wage gap is decomposed in three steps. First, I allow men and women to have the same child-related interruptions, while keeping equilibrium wages and job allocation decisions fixed. The red and yellow dotted lines show the resulting wage gaps after (1) equalizing parental leave duration, and (2) equalizing separation rates in addition to parental leave, respectively.[37] Since equilibrium effects are not considered at this point, the wage changes after equalizing parental leave can be interpreted as direct effects of experience gains (losses) of women (men). Greater job experience not only leads

[37] Instead of women taking 18 months and men 2 months, they each take 10 months in the counterfactual so the total number of PL months remains the same as before. The counterfactual separation rate is chosen somewhere in between the men and women's E-to-U rates, such that the total measure of employed workers is the same across the counterfactuals.

to higher wages in the same job, but also qualifies the worker for better positions on the ladder due to PAM, further boosting up wages for women (and lowering those for men). Equalizing separation rates can have two direct effects. First, women stay longer on the job with a reduced separation rate, and gain more human capital that leads to higher wages. Second, women fall off the job ladder less often, so they can now extract more rents through re-negotiations and on-the-job search, and advance more on the career ladder.

FIGURE 8. Gender wage gap decomposition



NOTES: The lines represent the log wage gap between men and women over the life-cycle. The top black solid line is the wage gap based on model estimates. The colored lines are the counterfactual wage gaps after adding one channel at a time: (1) Equal PL duration by gender, without changing equilibrium wages and job allocations. (2) Add equal separation rates, without equilibrium effects. (3) Implement the new equilibrium job allocations implied by equal PL and separations. (4) Implement new equilibrium wages. (5) Same preference for amenities by gender in addition to (1)-(4).

Parental leave and job separations channels both have large direct effects on the gender wage gap, particularly in later stages of a career. These channels together lead to only a 1.7 log point reduction in the wage gap during the early career phase (years 1 to 3), as educated men and women behave similarly before having children. However, the direct effects of changes in labor force attachment accumulate over time. By late career (16 to 24 years post-graduation), these channels contribute to a substantial portion of the wage gap, accounting for 11 out of 15 log points.

The second step of the decomposition is to measure the effects of child-related interruptions on (3) equilibrium job allocations and (4) equilibrium wages. When parental leave durations and separation rates are equalized between men and women, employers who anticipate similar behaviors of male and female workers around childbirth would adjust decisions on both hiring and wage margins.

In order to measure changes in equilibrium job allocations, I allow match formation and mobility to change to the new equilibrium while keeping wage policies the same as in the old equilibrium. In the new equilibrium, jobs in the highest productivity category that did not hire low-HC women now start matching with both men and women in NC stage. High-HC men who did not accept low-end jobs in YC stage now start taking them. Even though match formation decisions only change for a handful of types of workers and firms, changes in job allocations would propagate to the rest of the distribution. Having more women in top jobs implies that some men would be "pushed" to lower jobs. Vice versa, more men being drawn to bottom jobs means women will contact these vacancies with lower probability and encounter vacancies elsewhere with a relatively higher probability. These changes in allocations, however, have only a small impact on the overall gender wage gap (see Table L1). They lead to an additional 0.4 to 0.8 log points' reduction in the gender wage gap. The small effect might be driven by the fact that allocation changes only occur for a small group of people, who do not influence average wages considerably. Another reason might be that wages are kept to the previous equilibrium where there are still substantial wage discounts against women especially at top firms.

Next, I implement new equilibrium wages under equal PL and separations in addition to the new equilibrium job allocations. Employers' differential wage offers to men and women are responsible for a substantial portion of the wage gap in early career, accounting for 3.7 out of 9.1 log points during the first 3 years post-graduation. The equilibrium wage effects dissipate over time as more and more workers move beyond child-rearing ages, although the human capital effects from earlier job allocations are carried over to infertile ages.

In the third step, I compute a new equilibrium based on equal valuations of family-friendly amenities between men and women,[38] in addition to equal parental leave and separation rates. There are both wage and mobility changes in the new equilibrium, and altogether these changes explain an additional 1.5 to 1.2 log points of the gender wage gap in mid- and late-career, respectively. Since men and women have very similar values for amenities in the *No Child* stage, preference for job amenities explains little of the wage gap in early career.

[38] I set women's preference equal to men's before childbirth, and do not allow their preferences to change after childbirth.

Table L4 conducts a different counterfactual. It shows the effect of one channel at a time, while keeping the other channels fixed to the old regime.[39] Notably, there are important interactions between the channels, as the combined effect of multiple channels often differs from the sum of their individual impacts. For example, when equal parental leave and separation rates are combined with the new equilibrium job allocations (while maintaining the previous wage policies), the joint effect on the wage gap is more pronounced. This is because the advantages of increased labor force attachment for women are enhanced by the greater availability of higher-level job opportunities.

It may seem counterintuitive that the combined effect with new equilibrium wages, in contrast, is smaller in magnitude than the sum of the individual impacts. This occurs because changes in equilibrium wage offers vary across different job types and life stages. As we transition from a system with unequal parental leave and E-to-U rates to one with equal conditions, wage offers to women typically increase in the *No Child* stage but may decrease in some worker-job matches in the *Young Child* stage. This happens because women are willing to accept lower wages in some positions in return for the improved job prospects in YC stage in the new equilibrium.

5.2 Under-representation of women in top positions

While wages are a key focus in discussions about gender inequality, another important question is why so few women reach top-level positions compared to men. To what extent is the "glass ceiling" shaped by individual choices regarding labor supply and job amenities, versus firms' decisions regarding job allocations for men and women?

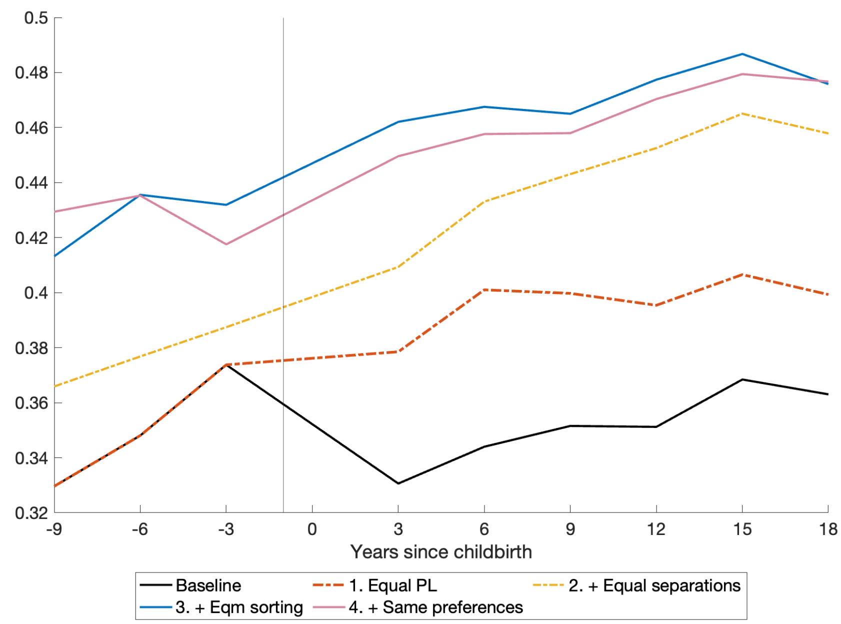
Figure 9 addresses the question by investigating the share of women in the most productive jobs, which are jobs in categories 6 and 7 in the model that correspond to mostly management and professional positions in the data. In the estimated model, women's representation in top positions rises slowly from 33 to 37% in the nine years leading up to childbirth. However, this percentage declines immediately after the birth of the first child, as depicted by the bottom black solid line of Figure 9 (see also the related table Table L3).

Similar to the decomposition in Section 5.1, I proceed in 3 steps. First, I eliminate gender differences in labor force attachment without changing the equilibrium job allocations. The red dotted line in Figure 9 illustrates that equalizing parental leave increases the proportion of women in top positions by approximately 5 percentage points over the

[39] For example, the row "(4) Equilibrium wages" shows the effects of new equilibrium wages on the gender wage gap, while fixing PL and separation parameters to the old (unequal) rates, and keeping equilibrium allocations to the old equilibrium.

ten years following childbirth. The region between the red and black lines reflects a direct effect of human capital gains (losses) of women (men). The model estimates imply that there are strong production complementarities between worker skills and job productivities, so the forces of PAM would push women to highly-productive jobs as their human capital improves.

FIGURE 9. Counterfactual female shares in top jobs around childbirth



NOTES: The lines represent the share of women in top jobs (top two job productivity categories), by years since first childbirth. The bottom black solid line is the female share implied by model estimates. The colored lines are counterfactual female shares when additional channels are added sequentially: (1) Equal PL duration by gender, without equilibrium effects. (2) Add equal separation rates by gender, without equilibrium effects. (3) Implement the new equilibrium allocations implied by equal PL and separations. (4) Same preference for amenities by gender in addition to (1)-(3).

Similarly, equalizing E-to-U rates also boosts the female share in top positions, by an amount equivalent to the region between the yellow and red dotted lines. With increased job experience and fewer career interruptions, women advance more into management through assortative matching and on-the-job search. Their representation in top roles increases by about 3 percentage points before childbirth and nearly 6 percentage points fifteen years after childbirth. Together, these two labor force attachment channels alone could eliminate over half of the gender imbalance in top positions following childbirth. In essence, most top positions are open to both men and women in the baseline economy, but fewer women with the requisite accumulated experience are available compared to men.

Second, I implement the new equilibrium job allocations and sorting after equalizing parental leave and separation rates. As parental leave becomes shorter and job spells more stable for women (and the opposite for men), employers and workers might make different match formation decisions. The blue solid line in Figure 9 shows the resulting female share in top positions as some jobs change their equilibrium hiring policies. In the *No Child* stage, certain top jobs that were previously inaccessible to young women in the baseline economy now begin to match with them in the new equilibrium. These highly-productive positions provide significant learning opportunities in early career stages, and women's human capital grows at an overall faster rate than in the baseline economy. Moreover, as women anticipate a less interrupted career going forward, they also shift their job search towards better positions. As a result, high-skilled women start to reject more low-end jobs, further enhancing their human capital.

Better job opportunities in the *No Child* stage also affect choices after having children. Women's improved skills and career prospects make them less likely to (endogenously) quit their jobs after parental leave, and this reduced likelihood of quitting has lasting positive effects on their career trajectories for decades. The job allocations channel primarily improves women's representation in top jobs in the period before childbirth, increasing it by 4 to 6 percentage points over the ten years prior to birth. Even though employers' job allocation decisions during the non-fecund stage remain the same as in the baseline economy, the initial access to top jobs has persistent effects in the long run – it increases women's representation in top managerial jobs by 4 to 2 percentage points five to fifteen years after childbirth.

Third, I compute the new equilibrium implied by equal amenity preferences across genders. As women's valuation for family-friendly amenities is reduced to men's level, it moderately decreases the proportion of women in top positions by 1 percentage point both before and after childbirth. This is because jobs in the top categories that provide flexibility and other amenities now become less attractive to women, and some women in these jobs might switch to lower-ranked jobs that provide less amenities but pay more. Therefore, equalizing men and women's preferences slightly exacerbates women's underrepresentation in top jobs, but improves the gender wage gap (as shown in Figure 8).

5.3 Counterfactual policy experiments

The section below considers three counterfactual policies that aim to reduce gender gaps in the labor market – a "daddy month" parental leave expansion, equal hiring in top jobs, and an equal pay policy. I compute the new equilibrium and quantify the effects of each policy on the gender wage gap as well as the female share in top positions over the life-cycle.

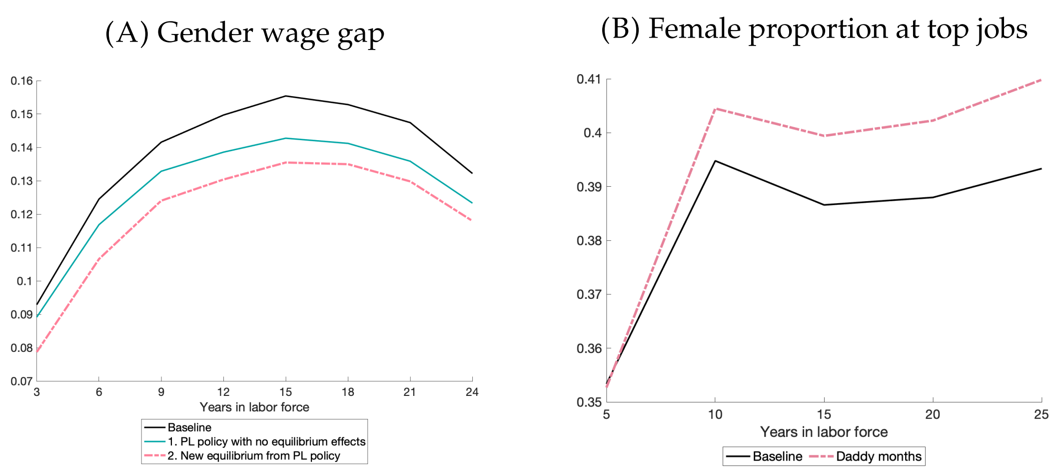
5.3.1 Daddy months

In Finland and many other Nordic countries, there is generous wage-replaced parental leave of durations from 6 months to over a year. Leave days could be split evenly between the parents, but it is almost always the mother who takes up most if not all of the shared leave. Many of these countries have then introduced 1 to 3 months of "daddy months" to encourage fathers to spend more time with the baby (Dahl, Løken and Mogstad, 2014).

I consider a counterfactual where fathers' leave increases by 2 months per child while mother's parental leave decreases by 2 months. To do this, I calibrate the parental leave exit shocks η_m and η_f so that men's leave duration per child increases from 2 to 4 months on average, while that of women's decreases from 18 to 16 months.

The daddy month reform is quite effective in reducing the gender wage gap throughout the life-cycle. As shown in Figure 10(A), the wage gap closes by 15% during the first 3 years of working, and over 10% afterwards. About half of the impact on wages comes from a change in equilibrium wage offers during pre-child years. Even though the shift of two months is not enough to change the job allocation decisions in years prior to childbirth, women's wages are now closer to men's when they are hired. Women also gain more human capital during mid-career because they return to work sooner after having children, while men accumulate less. This slightly balances the gender ratio in top jobs as the proportion of women increases from 39 to 41 percent by year 25 (see Figure 10(B)).

FIGURE 10. Counterfactuals under daddy months policy



One caveat of this policy is that it might not result in a pareto improvement – the progress in women's careers might come at the expense of men's. In order to assess the overall social value of the policy, define social welfare (SW) as the sum of the production of the employed matches and the home production of the unemployed net of the total cost of vacancies:

$$SW = \sum_{g,t,\boldsymbol{x}} b\,k \cdot u_t^g(\boldsymbol{x}) + \sum_{g,t,\boldsymbol{x},\boldsymbol{y}} f(k,p) \cdot h_t^g(\boldsymbol{x},\boldsymbol{y}) - \sum_{\boldsymbol{y}} c \cdot v(\boldsymbol{y}).$$

By the time men become fathers, they are already in slightly more advanced positions than women and are producing more output, so the output loss of having men spend 2 months at home cannot be fully compensated by output gains of women working 2 months more. However, the net loss in social welfare is very small (only 0.02% of total welfare). Also, paying men on parental leave is more costly since the benefits are proportional to wages and men typically earn more than women. In order to fund the new policy, the tax rate on flow output increases modestly from 2.80% to 2.88%.

This counterfactual assumes full take-up of earmarked paternity leave, which is reasonable given that the division of parental leave within households has historically responded strongly to policy design. In Finland, parental leave has remained highly skewed toward mothers despite the option of equal sharing since 1985. When no leave was earmarked, men took very little. When two months were reserved for fathers during the sample period, take-up clustered at that level, suggesting that men tend to take the minimum leave possible without forfeiting benefits.

5.3.2 Equal hiring policy in top jobs

To address the under-representation of women in top-earning jobs, many countries have passed legislature to require a certain percentage of female board members in public companies. Finland requires state-owned enterprises to reserve 40% of board seats to female directors. However, the evidence on the effectiveness of these policies in reducing gender gaps is mixed at best (Bertrand, Black, Jensen and Lleras-Muney, 2018).

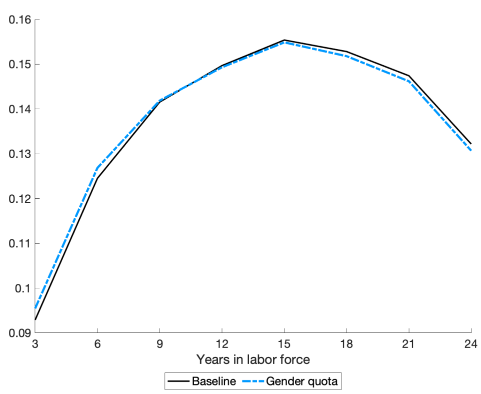
There is no direct way of implementing a gender quota in the model since the proportion of women in a particular job category depends not only on the optimal hiring rule of the job, but also on the transition rates and workers' mobility to all other jobs in equilibrium. In practice, I implement an "equal hiring" policy that requires the top jobs (those in the highest productivity category) to have the same hiring rule towards a woman and a man of the same $\boldsymbol{x}$ type.

The policy essentially changes hiring rules of top positions towards low-HC women in the *No Child* stage. Since these matches would not have been formed in the absence of the equal hiring policy, there is no standard wage protocol about how to split the (negative) match surplus. In this exercise, I assume that the employer sets the wage to cover the vacancy value of the job, and the worker gets the rest of the match value.

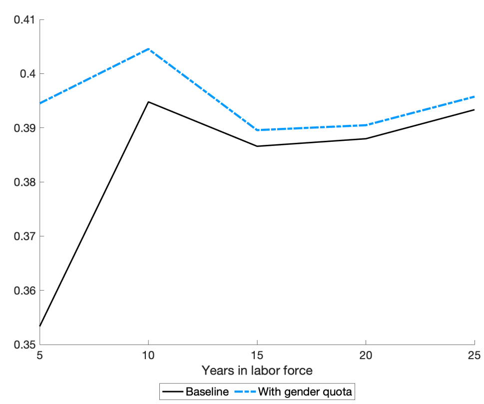
Unsurprisingly, banning hiring discrimination at top jobs improves women's representation in those jobs during the early years of workers' professional lives. Figure 11(B) shows that the female share increases from 35.5 to 39.5 percent in top jobs during the first 5 years of work. However, this effect is very short-lived. Since the equal hiring policy does not address child-related interruptions, women start falling behind men in human capital levels soon after childbirth, and are thus less likely to stay in highly productive jobs later on due to forces of PAM. The proportion female in top jobs almost falls back to baseline levels during child-rearing years. The overall effect of the policy on the share of women in top jobs is only slightly positive by the end of the life-cycle.

FIGURE 11. Counterfactuals under equal hiring policy

(A) Gender wage gap



(B) Female proportion at top jobs



Even though the equal hiring policy improves women's representation at top jobs, employers undo this policy by exerting more wage discounts. Women hired under the new policy receive lower wages than men in the same job during the early years of the life-cycle. This is because employers are now required to form matches with all women even though some matches generate negative surpluses; as a result, the new female hires have to "compensate" the employers by accepting sub-par wages. Since the new hires are a small proportion of the working population, the overall wage gap only increases by a small amount (by 3% in 6 years). However, being employed in high-productivity jobs in early career allows young women to gain skills at a faster rate, and the human capital gains more than compensate for the initial wage loss. Figure 11(A) shows that the negative impact of the policy on women's wages disappears nine years after labor market entry.

There is a small welfare gain when women gain access to top jobs. This is because when employers make hiring decisions, they only care about maximizing their individual profits and do not take into account the social benefit of upgrading workers' human capital. The net welfare gain from the equal hiring policy is 0.01%.

5.3.3 Equal pay policy

Many OECD countries have passed some form of Equal Pay Act that requires men and women in the same workplace be given equal pay for equal work. The Finnish Equality Act requires companies with 30 or more full-time employees to draft a gender equality plan, which should include an assessment of pay differences between men and women who perform work of equal value.[40]

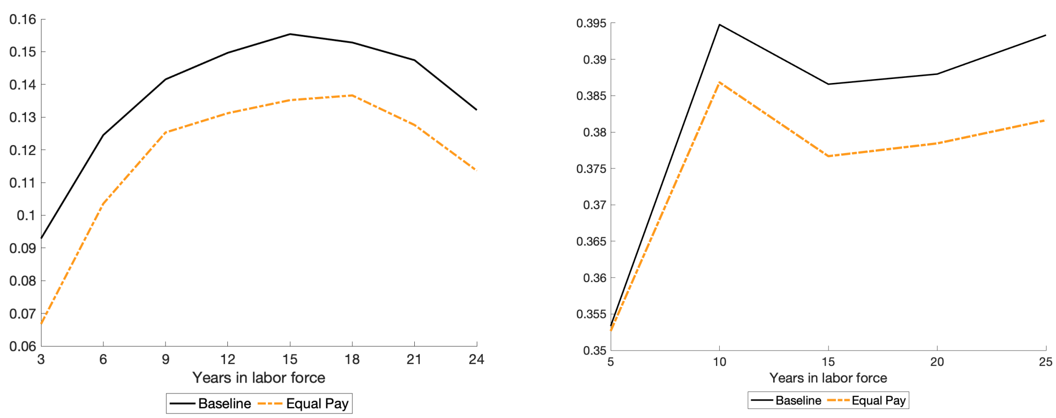
In the equal pay counterfactual, I require men and women of the same $\boldsymbol{x}$ type working in the same $\boldsymbol{y}$ job to receive the same starting wage. I compute the equivalent lifetime value of the female worker $W_{0t}^{f}(\boldsymbol{x}, \boldsymbol{y})$ implied by having men's flow wages $\omega_{0,t}^{m}$ in each age segment t, and re-calculate employer's share in the surplus:

$$\underbrace{\Pi_t^f(\omega_{0,t}^m, \boldsymbol{x}, \boldsymbol{y}) - J(\boldsymbol{y})}_{\text{employer's share}} = S_t^f(\boldsymbol{x}, \boldsymbol{y}) - \underbrace{\left(W_t^f(\omega_{0,t}^m, \boldsymbol{x}, \boldsymbol{y}) - U_t^f(\boldsymbol{x})\right)}_{\text{worker's share}}.$$

When the worker's value W_{0t}^{f} is required to increase, the employer's portion might become negative, in which case the match would dissolve.

FIGURE 12. Counterfactuals under equal pay policy

(A) Gender wage gap (B) Female proportion at top jobs



I simulate the workers' careers with the equal wage policy, allowing matches to dissolve if the employer's value $\Pi_t^g(\omega_{0,t}^m, \boldsymbol{x}, \boldsymbol{y})$ fall below the vacancy value $J(\boldsymbol{y})$. Figure 12 shows that the equal pay policy unsurprisingly reduces the gender wage gap by 2 to 4 log points. However, some matches are no longer sustained in the periods after having children. As a result, women are more likely to be unemployed and fall off the career

[40]Details of the Equality Act and related reforms can be found at:
https://www.finlex.fi/en/laki/kaannokset/1986/en19860609

ladder, although the effect size is very small. Figure 12(B) shows that the proportion of women in top jobs decreases by one percentage point in years 15-25.

6 Conclusion

This paper studies the mechanisms underlying gender gaps over the life-cycle — workers' human capital accumulation, preference for amenities, and employers' wage-setting and job allocation decisions. I propose an equilibrium search model with capacity constraints, production complementarities, fertility and parental leave, and taste for job amenities. The model is estimated using matched employer-employee data from Finland combined with occupation-level data on amenities from the Finnish Quality of Work Life Survey.

Men and women behave very differently in the labor market especially after having children. Employers take into account these gender differences and allocate different jobs to men and women even before they have children. The parameter estimates of the model imply that a sizable portion of the gender wage gap in early career (4.0 out of 9.1 log points) can be attributed to employers' differential sorting and wage-setting by gender based on fertility-related concerns. In late career stages, gender disparities in labor force attachment account for the majority of the wage gap (11 out of 15 log points).

The most effective policies for reducing gender gaps are those that alleviate women's family responsibilities, such as expanding childcare options to lower women's separation rates and increasing parental leave for fathers. These measures would not only help women gain more on-the-job experience but also shift firms' expectations, leading employers to reduce wage markdowns and improve job opportunities for women. In contrast, policies aimed at eliminating hiring discrimination in top jobs might reduce women's wages in early-career, while equal pay policies can lower the proportion of women in top positions as employers adjust their hiring strategies.

Overall, the policy counterfactuals suggest that it might be difficult to achieve gender equality in the workplace without greater equality in family responsibilities, given the sizable effects of employers' equilibrium decisions. Requiring equality in one margin (either wages or job allocations) often prompts firms to counteract the policy on the other margin. Requiring equality across both margins could be challenging and costly to enforce, as forward-looking, rational employers have an inherent incentive to deviate from such mandates.

Gender differences in parental leave duration and job separation rates are treated as exogenous in this paper. Studying households' joint optimization over these decisions would be a natural extension, but is beyond the scope of the current analysis. In practice, the division of child-rearing responsibilities is largely shaped by prevailing gender norms and family leave policies rather than by career concerns alone, making it reasonable to take these differences as given. An extension of the model might involve formalizing intra-household decisions, where spouses jointly choose their parental leave lengths and separation rates while taking into account their labor market prospects. Employers' priors that women are more prone to career interruptions might become a self-fulfilling prophecy if the resulting wage and job opportunity discrimination leads women to specialize in household production. Conversely, policies that promote gender equality in the labor market may trigger social norm changes and considerable feedback effects if households respond by sharing family responsibilities more equally. Therefore, increased equality in the labor market may reinforce gender equality within the household, and vice versa. Quantifying the long-run impacts of such propagating effects is left for future research.

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Appendix

Appendix A Data description and sample selection

The Finnish Longitudinal Employer-Employee Data (FOLK) is assembled by Statistics Finland from numerous administrative registers, and covers the entire resident population aged 15 to 70 between years 1988 and 2016. FOLK provides detailed employment histories for each worker. Using the start and end dates of each employment relationship, I create a monthly employment status for each worker – employed, unemployed, or on parental leave. Since FOLK can be linked to the official population register, I also observe the birth date of each child of the worker and use it to infer the worker's parental leave status when he/she starts collecting benefits around that date.

The hourly wage data comes from the Structure of Earnings Statistics (SES). The SES consists of large-scale surveys collected by the Employers' Association in the last quarter of each year from 1995 to 2013. It covers all public sector workers and 55 to 75 percent of private sector workers depending on the year. The following groups in the private sector are either entirely excluded or at least severely under-represented: 1. small (less than 5 persons) enterprises; 2. the vast majority of non-organized (mainly small) enterprises; 3. agriculture, forestry and fisheries; 4. international organizations; 5. company management and owners and their family members; 6. the employment relationships beginning or ending during the reference month.

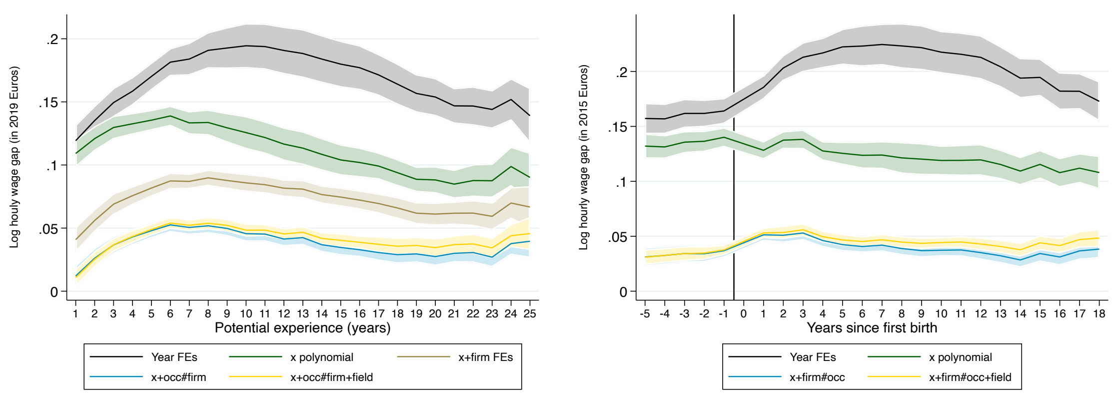
SES observations are on the yearly level (as opposed to daily in FOLK), and some firms might not be surveyed by SES in certain years. In the estimation, I use sample weights in the simulations to account for potential missing data from small firms. Wages and occupations are observed in the SES once a year from 1995 to 2013 in the last quarter of the calendar year. If the worker has wages from more than one employer in a quarter, I keep only the wage from the "main" job – the full-time job if there is one, or the job with the most earnings if all jobs are part-time. I trim the top 0.5% of the wage distributions in each year, which tend to be very thin and cover wide ranges. I remove macroeconomic fluctuations in wages and transition rates by taking out year fixed effects in all moments calculations.

I drop workers whose age is in the bottom or top 5 percentiles of the age distribution at graduation, so that workers in my sample are aged between 24 and 31 when they graduated master's. I drop small firms that have never had more than 2 workers during the sample period. I only include periods after the individuals have completed their master's education. Unemployment of 2 months or less is counted as the final tenure of the previous spell. Similarly, employment of 2 months or less is counted as non-employment. After sample selection, I have an unbalanced panel of 116,781 workers, and 25,951 distinct firm-occupations over the course of 18 years.

Appendix B Gender differences in the labor market

FIGURE B1. Gender wage gap decomposition

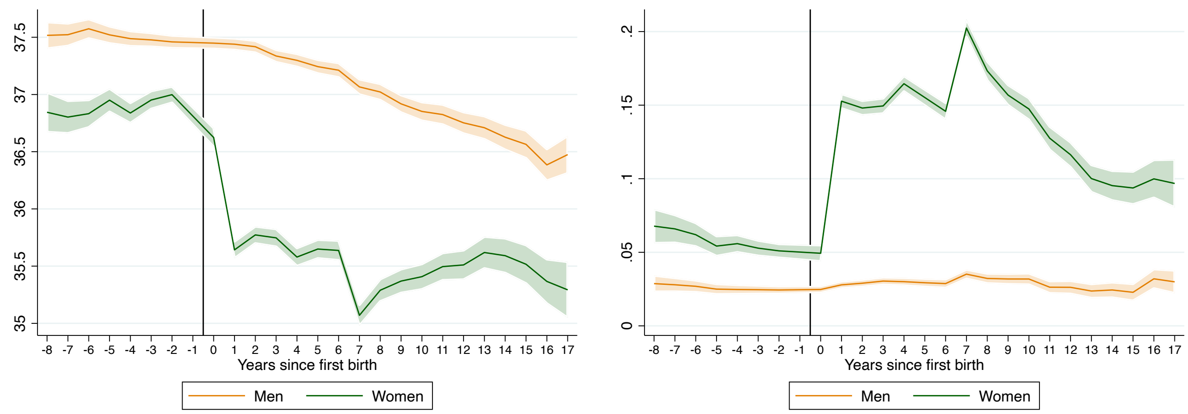
(A) Over the life-cycle (B) Around first childbirth



NOTES: The lines represent the coefficients on the male dummy interacted with potential experience or years since childbirth. Shaded areas represent 95% confidence intervals. Standard errors are clustered at the firm level. The coefficients are obtained from regressions of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience (x) in addition to (i); (iii) firm fixed effects in addition to (i) and (ii) (I forgot to do this for around childbirth); (iv) a full set of interactions of firm and occupation dummies in addition to (i) and (ii); and (v) 3-digit field of study dummies in addition to (i), (ii) and (iv).

FIGURE B2. Work hours around birth

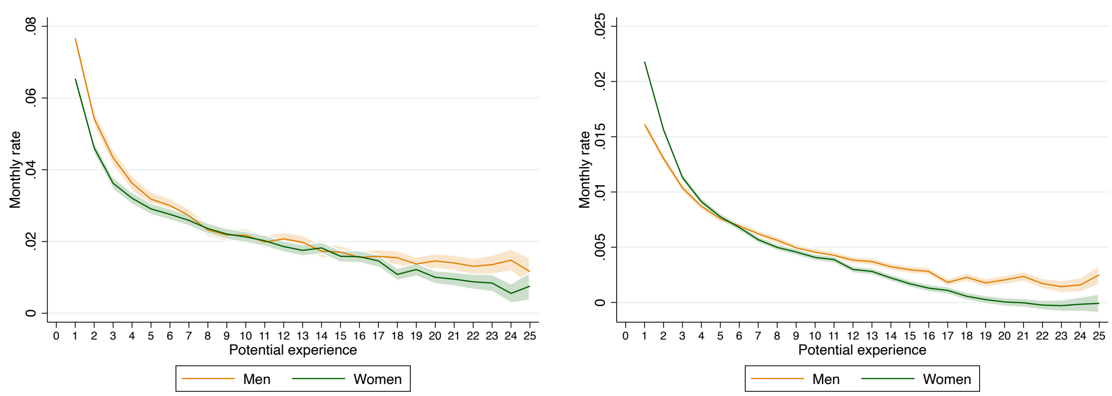
(A) Contracted weekly hours (B) Proportion with part-time status



NOTES: The lines represent the coefficients obtained from regressions of outcome variables on the number of years since first birth, separately for men and women, with individual fixed effects and calendar year fixed effects. Shaded areas represent 95% confidence intervals.

FIGURE B3. Transitions over the life-cycle

(A) U-to-E transition rate (B) Employer-to-employer transition rate



NOTES: The lines represent the coefficients obtained from regressions of outcome variables on potential experience, separately for men and women. Shaded areas represent 95% confidence intervals.

Appendix C Family-friendly amenities

I use several data sources to construct the amenity measure. The Finnish Quality of Work Life (QWL) Surveys are extensive studies of a representative sample of 4000 to 6000 wage or salary earners in Finland in each wave 1977, 1984, 1990, 1997, 2003, 2008 and 2013. It documents how people feel about their working conditions related to physical or social environment, job satisfaction, work orientation and so on. QWL surveys ask questions related to flexibility (positive amenities) and over-working (negative amenities), listed below:

Flexibility:

- Have you agreed with the employer to work occasionally at home?
- Can you influence starting and finishing times for your work by at least 30 minutes?
- Can you use flexible working hours sufficiently for your own needs?
- Do have the possibility for brief absences from work in the middle of the working day to run personal errands?

Overwork:

- Do you sometimes work overtime without compensation?
- Have you been contacted about work outside of working hours during the last two months?
- Do you have to do more overtime work than you would like to?

The "family friendly" amenity index of each job is the first principal component of the above 7 QWL questions aggregated at 2-digit occupation level,[41] actual hours worked from the labor force survey, and the opportunity to do part-time work in a firm-occupation cell from SES.

Appendix Table C1 shows the factor loadings of these variables and the proportion of variation not explained by the first component. The amenity index is largely driven by the variables related to hours, as the first principal component loads more heavily on the QWL measures on overwork than on the measures of flexibility (in absolute value).[42]

[41] 2-digit occupations may not be detailed enough to give a fully comprehensive picture. However, the 2013 QWL is the only wave that provides information on the 3-digit level, and one cannot go into more detailed occupations due to the sample size of the surveys.

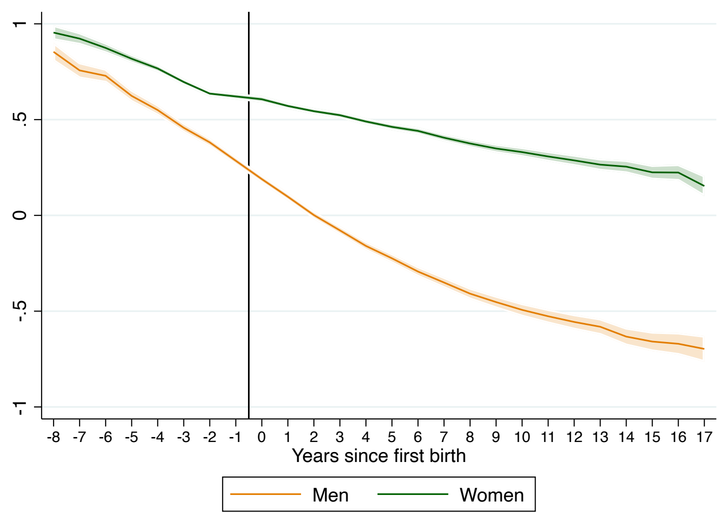
[42] The loadings of the flexibility measures are negative, possibly because highly flexible occupations (e.g. managers, science and engineering professionals etc.) also have high overtime requirements, and the index aligns more with overtime. It would be interesting to analyze how workers trade off the two dimensions of family-friendly amenities (high flexibility and low overtime hours) in future work.

TABLE C1. Principal component analysis for the amenity index

Variables	Factor loading	Unexplained proportion
1. Work from home	-0.315	.522
2. Flexible start/end	-0.378	.312
3. Flexible hours	-0.256	.683
4. Run errands during work	-0.338	.450
5. Overtime without pay	0.369	.344
6. Contacted after work	0.403	.217
7. Too much overtime	0.375	.322
8. Actual hours worked (LFS)	0.347	.419
9. Proportion part-time	0.140	.905

NOTES: The table shows the factor loading of each variable for the first principal component. Negative amenities (variables 5 to 8) are multiplied by -1 before entering the principal component analysis, so all the variables can be interpreted as good amenities.

FIGURE C1. Amenity index of workers' jobs around childbirth



NOTES: The lines represent the coefficients obtained from a regression of the (continuous) amenity index on the number of years since first birth, separately for men and women, with worker fixed effects. Shaded areas represent 95% confidence intervals.

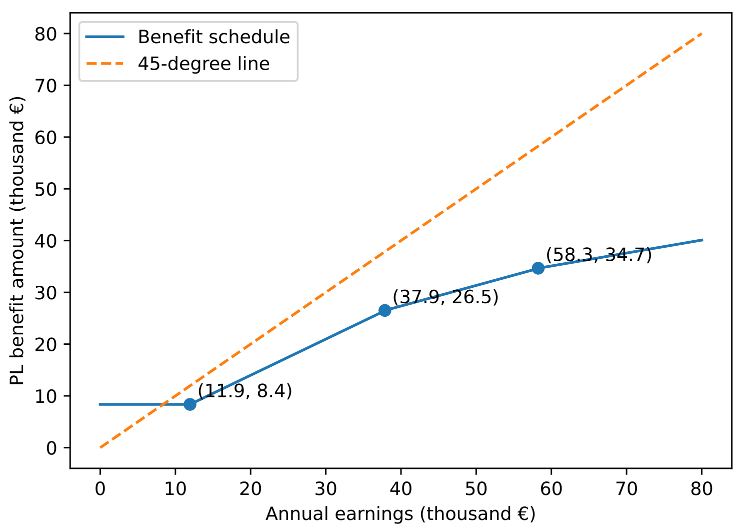
The continuous amenity index at the firm-occupation level is used to classify jobs into three discrete types (along the α dimension) in Section 3: very high (more than 1 s.d. above the mean), high (between 0.5 and 1 s.d. above the mean) and regular jobs (the remaining). The empirical distribution of jobs across both productivity and amenity dimensions is shown in Figure 5.

Appendix D Parental leave system in Finland

The Finnish maternity allowance system was first introduced in 1964. During the sample period (1995-2013), parents are entitled to wage-replaced leave for a total of 12 months, in which 4 months are reserved for mothers, 2 months for fathers, and 6 months can be shared between the spouses. In addition, parents are entitled to Child Home Care Allowances until the child turns 3 years old. Both biological and adoptive parents are entitled to parental leave on the basis of permanent residence in Finland.

Finnish law provides strong job protection for workers on parental leave. Under the Employment Contracts Act (55/2001, Chapter 4, Section 9), employers are required to hold the position open for the entire duration of the leave. Upon return, the worker is entitled to their former position on the same terms. If that is no longer possible, the employer must offer equivalent work with comparable duties, pay, and rank.[43]

FIGURE D1. Parental leave allowances pay schedule (2019 euros)



The parental leave benefit is a piecewise linear function of tax verified earnings in the previous calendar year, as shown in Figure D1 (in 2019 euros). Workers with annual earnings below €11,942, including the non-employed, receive a flat minimum benefit of €8,358 per year. For earnings between €11,942 and €37,861, the benefit equals 70% of annual earnings. The marginal replacement rate then steps down to 40% for earnings between €37,862 and €58,252, and further to 25% for earnings above €58,252. In my sample, workers receive on average 63% of their prior earnings in parental leave benefits.

[43]Dismissal during parental leave is heavily restricted. Termination is legally presumed to be based on the leave unless the employer can prove otherwise, and is only permitted if the employer's operations cease completely.

After the parental leave is over, parents can continue to care for the child at home and receive the Home Care Allowances (HCA). The HCA may be paid to either parent, although it is predominantly the mother who takes up the allowance. The HCA benefit amount consists of two parts – there is a fixed amount of 338.34 euros per month for one child under 3, and a means-tested amount targeted at low-income families up to 180 euros per month. In addition, there is sibling extra and municipality-based supplements. For details of HCA, please refer to Kosonen (2014).

The benefit amount of the parental leave allowance and the HCA claimed are separately reported in the FOLK data for each individual in each calendar year. This paper uses the pay schedule in Figure D1 and the fixed HCA amount adjusted by inflation to infer the total number of months of parental leave taken for each worker.

Since I observe the exact amount of parental leave benefits collected around the time of childbirth, I can pinpoint the month at which the worker stops collecting benefits. If a worker is not associated with an employer and is not collecting parental leave benefits in a particular month, he/she is considered to be unemployed.[44] According to this measure, female separation rate is already a little higher than male's prior to birth, but the big difference appears right after childbirth, where women's separation spikes and remains well above men's for many years after childbirth (Figure 4(B)).

[44] If someone is unemployed for only two months or less after she stops collecting parental leave benefits, I consider it as measurement error in leave duration calculations and do not count the months as unemployment. A separation is only indicated for unemployment of 3 months or more.

Appendix E Wage determination and workers' values

Recall from equation (1) that $W^g_{0t}(\boldsymbol{x},\boldsymbol{y}) = U^g_t(\boldsymbol{x}) + \beta\, S^g_t(\boldsymbol{x},\boldsymbol{y})$. Given equilibrium values U_t, S_t, the equation below illustrates an example of the worker's value W_{0t} when he/she receives a wage ω_0 out of unemployment in the *No Child* stage:

$$\begin{aligned}\Big(r + H_e(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, W^g_{0,NC}(\boldsymbol{x},\boldsymbol{y}) = {}& \omega^g_{0,NC}(\boldsymbol{x},\boldsymbol{y}) + q^g(\epsilon,\alpha) + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\, \widetilde{W}^g_{0,NC}(\boldsymbol{x}',\boldsymbol{y}) \\ & + \delta^g_{NC}\, U^g_{NC}(\boldsymbol{x}) + \chi\, \widetilde{W}^g_{0,PL}(\boldsymbol{x},\boldsymbol{y}) + \gamma\, \widetilde{W}^g_{0,NF}(\boldsymbol{x},\boldsymbol{y}) \\ & + s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}')\, \mathbb{1}[S' \le S] \cdot \Big[W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) - W^g_{0,NC}(\boldsymbol{x},\boldsymbol{y})\Big]^+ \\ & + s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}')\, \mathbb{1}[S' > S] \cdot \Big[W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') - W^g_{0,NC}(\boldsymbol{x},\boldsymbol{y})\Big]^+ \end{aligned} \tag{13}$$

where the total rate of HC change is denoted by $H_e(k,p) = \sum_{k'} \rho_e(k'|k,p)$, and $[W]^+$ denotes $\max\{W,0\}$. When a worker's human capital changes from k to k' in the next period, the wage does not update until there is a credible outside option.

At any point in time, the match can dissolve endogenously if surplus falls below zero:

$$\widetilde{W}^g_{0,t}(\boldsymbol{x},\boldsymbol{y}) = \max\Big\{W^g_{0t}(\boldsymbol{x},\boldsymbol{y}),\ U^g_t(\boldsymbol{x})\Big\}, \qquad t = \{NC, PL, YC, NF\}$$

When workers move from job $\boldsymbol{y}$ to $\boldsymbol{y}'$, their flow wage at the new job is ω_1 such that they get a value of $W_{1t}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')$. An example in NC stage follows:

$$\begin{aligned}\Big(r + H_e(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') = {}& \omega^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') + q^g(\epsilon,\alpha) + \delta^g_{NC}\, U^g_{NC}(\boldsymbol{x}) \\ & + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y}')\, \max\Big(\widetilde{W}^g_{1,NC}(\boldsymbol{x}',\boldsymbol{y},\boldsymbol{y}'),\ \widetilde{W}_{0,NC}(\boldsymbol{x}',\boldsymbol{y}')\Big) \\ & + \chi\, \widetilde{W}^g_{1,PL}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') + \gamma\, \widetilde{W}^g_{1,NF}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' \le S'] \cdot \Big[W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}'',\boldsymbol{y}') - W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')\Big]^+ \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' > S'] \cdot \Big[W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}'') - W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')\Big]^+ \end{aligned} \tag{14}$$

Similarly, when a poaching job $(\boldsymbol{y}')$ triggers a wage renegotiation but not a job-to-job transition (when $S' < S$), the renegotiated wage ω_2 is such that:

$$\begin{aligned}\Big(r + H_e(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) \;=\; & \omega^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) + q^g(\epsilon,\alpha) \\ & + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y}) \,\max\Big(\widetilde{W}^g_{2,NC}(\boldsymbol{x}',\boldsymbol{y}',\boldsymbol{y}),\; \widetilde{W}_{0,NC}(\boldsymbol{x}',\boldsymbol{y})\Big) \\ & + \delta^g_{NC}\, U^g_{NC}(\boldsymbol{x}) + \chi\, \widetilde{W}^g_{2,PL}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) + \gamma\, \widetilde{W}^g_{2,NF}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' \le S] \cdot \Big[W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}'',\boldsymbol{y}) - W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})\Big]^+ \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' > S] \cdot \Big[W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}'') - W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})\Big]^+ \end{aligned} \tag{15}$$

Appendix F Value functions

The values of the non-employed workers in PL, YC and NF stages are:

$$(r + \eta^g + \gamma)\, U^g_{PL}(\boldsymbol{x}) = b(k) + \eta^g\, U^g_{YC}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) + \sum_{k'} \rho_u(k'|k)\, U^g_{PL}(k',\epsilon) \tag{16}$$

$$\begin{aligned}(r + \chi + \gamma)\, U^g_{YC}(\boldsymbol{x}) = {} & b(k) + \chi\, U^g_{PL}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) \\ & + \sum_{k'} \rho_u(k'|k)\, U^g_{YC}(k',\epsilon) + \sum_{\boldsymbol{y}} \lambda\, v(\boldsymbol{y})\; \beta\, [S^g_{YC}(\boldsymbol{x},\boldsymbol{y})]^+ \end{aligned} \tag{17}$$

$$(r + \phi)\, U^g_{NF}(\boldsymbol{x}) = b(k) + \sum_{k'} \rho_u(k'|k)\, U^g_{NF}(k',\epsilon) + \sum_{\boldsymbol{y}} \lambda\, v(\boldsymbol{y})\; \beta\, [S^g_{NF}(\boldsymbol{x},\boldsymbol{y})]^+ \tag{18}$$

The joint values of matches in *Young Child* and *Non-Fecund* stages are:

$$\begin{aligned}(r + H_e(k,p) + \delta^g_{YC} + \chi + \gamma)\, P^g_{YC}(\boldsymbol{x},\boldsymbol{y}) = {} & (1-\tau) f(k,p) + q^g_{YC}(\epsilon,\alpha) + \sum_{k'} \rho_e(k'|k,p)\, \widetilde{P}^g_{YC}(\boldsymbol{x}',\boldsymbol{y}) \\ & + \delta^g_{YC}\Big(J(\boldsymbol{y}) + U^g_{YC}(\boldsymbol{x})\Big) + \chi\, \widetilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) + \gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y}) \\ & + \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\; \beta\, [S^g_{YC}(\boldsymbol{x},\boldsymbol{y}') - S^g_{YC}(\boldsymbol{x},\boldsymbol{y})]^+ \end{aligned} \tag{19}$$

$$\begin{aligned}(r + H_e(k,p) + \delta + \phi)\, P^g_{NF}(\boldsymbol{x},\boldsymbol{y}) = {} & (1-\tau) f(k,p) + q(\epsilon,\alpha) + \sum_{k'} \rho_e(k'|k,p)\, \widetilde{P}^g_{NF}(\boldsymbol{x}',\boldsymbol{y}) \\ & + \delta\Big(J(\boldsymbol{y}) + U^g_{NF}(\boldsymbol{x})\Big) + \phi\, J(\boldsymbol{y}) \\ & + \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\; \beta\, [S^g_{NF}(\boldsymbol{x},\boldsymbol{y}') - S^g_{NF}(\boldsymbol{x},\boldsymbol{y})]^+ \end{aligned} \tag{20}$$

where $H_e(k,p)$ denotes the total rate of HC change $\sum_{k'} \rho_e(k'|k,p)$.

Appendix G Steady-state balance equations

In a stationary equilibrium, flows into and out of any worker-job match must balance. Gender subscripts are suppressed for ease of exposition. The equations are synonymous for men and women.

$$u_{NC}(\boldsymbol{x}) \cdot \Bigg[\underbrace{\chi + \gamma}_{\text{fertility, ageing}} + \underbrace{\lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \mathbb{1}\left[S_{NC}(\boldsymbol{x},\boldsymbol{y}) > 0\right]}_{\text{job acceptance}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_u(x'|x)}_{\text{HC changes}} \Bigg] \tag{21}$$

$$= \underbrace{\delta_{NC} \sum_{\boldsymbol{y}, S>0} h_{NC}(\boldsymbol{x},\boldsymbol{y})}_{\text{exog. job destruction}} + \underbrace{\sum_{\boldsymbol{x}'} \sum_{\boldsymbol{y}} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{NC}(\boldsymbol{x}',\boldsymbol{y}) \mathbb{1}\left[S_{NC}(\boldsymbol{x},\boldsymbol{y}) < 0\right]}_{\text{endogenous quits}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{NC}(\boldsymbol{x}')}_{\text{HC changes}} + \underbrace{\xi_0\,(\boldsymbol{x})}_{\text{new entrants}}$$

$$u_{PL}(\boldsymbol{x}) \cdot \left[\eta + \gamma + \sum_{\boldsymbol{x}'} \rho_u(x'|x) \right] = \delta_{YC} \sum_{\boldsymbol{y}, S>0} h_{PL}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{PL}(\boldsymbol{x}') + \chi \left[u_{NC}(\boldsymbol{x}) + u_{YC}(\boldsymbol{x}) \right] \tag{22}$$

$$u_{YC}(\boldsymbol{x}) \cdot \left[\chi + \gamma + \lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \mathbb{1}\left[S_{YC}(\boldsymbol{x},\boldsymbol{y}) > 0\right] + \sum_{\boldsymbol{x}'} \rho_u(x'|x) \right] \tag{23}$$

$$= \delta_{YC} \sum_{\boldsymbol{y}, S>0} h_{YC}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{x}'} \sum_{\boldsymbol{y}} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{YC}(\boldsymbol{x}',\boldsymbol{y}) \mathbb{1}\left[S_{YC}(\boldsymbol{x},\boldsymbol{y}) < 0\right] + \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{YC}(\boldsymbol{x}') + \eta \cdot u_{PL}(\boldsymbol{x})$$

$$u_{NF}(\boldsymbol{x}) \cdot \left[\phi + \lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \mathbb{1}\left[S_{NF}(\boldsymbol{x},\boldsymbol{y}) > 0\right] + \sum_{\boldsymbol{x}'} \rho_u(x'|x) \right] \tag{24}$$

$$= \delta_{NF} \sum_{\boldsymbol{y}, S>0} h_{NF}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{x}'} \sum_{\boldsymbol{y}} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{NF}(\boldsymbol{x}',\boldsymbol{y}) \mathbb{1}\left[S_{NF}(\boldsymbol{x},\boldsymbol{y}) < 0\right] + \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{NF}(\boldsymbol{x}') + \gamma \left[u_{NC}(\boldsymbol{x}) + u_{PL}(\boldsymbol{x}) + u_{YC}(\boldsymbol{x}) \right]$$

$$h_{NC}(\boldsymbol{x},\boldsymbol{y}) \cdot \Bigg[\delta_{NC} + \gamma + \chi + \underbrace{s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \mathbb{1}\left[S_{NC}(\boldsymbol{x},\boldsymbol{y}') > S_{NC}(\boldsymbol{x},\boldsymbol{y})\right]}_{\text{leave for better jobs}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_e(x'|x,y)}_{\text{HC changes}} \Bigg] \tag{25}$$

$$= \underbrace{\lambda\, v(\boldsymbol{y})\, u_{NC}(\boldsymbol{x})}_{\text{job acceptance}} + \underbrace{s\lambda\, v(\boldsymbol{y}) \sum_{\boldsymbol{y}'} h_{NC}(\boldsymbol{x},\boldsymbol{y}') \mathbb{1}\left[S_{NC}(\boldsymbol{x},\boldsymbol{y}') < S_{NC}(\boldsymbol{x},\boldsymbol{y})\right]}_{\text{poaching from other jobs}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{NC}(\boldsymbol{x}',\boldsymbol{y}) \mathbb{1}\left[S_{NC}(\boldsymbol{x},\boldsymbol{y}) > 0\right]}_{\text{HC changes}}$$

$$h_{PL}(\boldsymbol{x},\boldsymbol{y}) \cdot \left[\delta_{YC} + \eta + \gamma \right] = \chi \left[h_{NC}(\boldsymbol{x},\boldsymbol{y}) + h_{YC}(\boldsymbol{x},\boldsymbol{y}) \right] \tag{26}$$

$$h_{YC}(\boldsymbol{x},\boldsymbol{y}) \cdot \left[\delta_{YC} + \chi + \gamma + s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \mathbb{1}\left[S_{YC}(\boldsymbol{x},\boldsymbol{y}') > S_{YC}(\boldsymbol{x},\boldsymbol{y})\right] + \sum_{\boldsymbol{x}'} \rho_e(x'|x,y) \right] \tag{27}$$

$$= \lambda\, v(\boldsymbol{y})\, u_{YC}(\boldsymbol{x}) + s\lambda\, v(\boldsymbol{y}) \sum_{\boldsymbol{y}'} h_{YC}(\boldsymbol{x},\boldsymbol{y}') \mathbb{1}\left[S_{YC}(\boldsymbol{x},\boldsymbol{y}') < S_{YC}(\boldsymbol{x},\boldsymbol{y})\right] + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{YC}(\boldsymbol{x}',\boldsymbol{y}) \mathbb{1}\left[S_{YC}(\boldsymbol{x},\boldsymbol{y}) > 0\right] + \eta \cdot h_{PL}(\boldsymbol{x},\boldsymbol{y})$$

$$h_{NF}(\boldsymbol{x},\boldsymbol{y}) \cdot \left[\delta_{NF} + \phi + s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \mathbb{1}\left[S_{NF}(\boldsymbol{x},\boldsymbol{y}') > S_{NF}(\boldsymbol{x},\boldsymbol{y})\right] + \sum_{\boldsymbol{x}'} \rho_e(x'|x,y) \right] \tag{28}$$

$$= \lambda\, v(\boldsymbol{y})\, u_{NF}(\boldsymbol{x}) + s\lambda\, v(\boldsymbol{y}) \sum_{\boldsymbol{y}'} h_{NF}(\boldsymbol{x},\boldsymbol{y}') \mathbb{1}\left[S_{NF}(\boldsymbol{x},\boldsymbol{y}') < S_{NF}(\boldsymbol{x},\boldsymbol{y})\right] + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{NF}(\boldsymbol{x}',\boldsymbol{y}) \mathbb{1}\left[S_{NF}(\boldsymbol{x},\boldsymbol{y}) > 0\right]$$

$$+ \gamma \left[h_{NC}(\boldsymbol{x},\boldsymbol{y}) + h_{PL}(\boldsymbol{x},\boldsymbol{y}) + h_{YC}(\boldsymbol{x},\boldsymbol{y}) \right]$$

Appendix H Smoothing shocks

Since worker types and job types are discrete while surplus values are continuous, there is no guarantee that the equilibrium solution algorithm always converges. This is due to the possibility that two jobs may offer very similar surplus values for a given worker type, for example when one job provides lower productivity with greater amenities and another offers higher productivity with less amenities. An infinitesimal change in the match values might alter the ranking of these jobs, resulting in a discrete shift in workers' mobility and the steady-state flows across jobs. Similarly, when a match has a surplus value close to zero, a slight change in the value function could cause the match to shift from being unacceptable to acceptable, or vice versa.

In order to break the ties between any two jobs and smooth the match formation decisions, I introduce an i.i.d. shock z upon each job arrival. An unemployed worker accepts a job with surplus S if $S + z > 0$. Similarly, an employed worker with match surplus S moves to a new job with match surplus S' if $S' + z > S$.

Assume that the i.i.d. shock z follows a Logistic distribution with mean 0 and scale parameter ζ, so its CDF $\Gamma(z) = \frac{1}{1+e^{-z/\zeta}}$. Then the expected value from a job with surplus S has an analytical expression:

$$\begin{aligned}
&\int (S+z) \cdot \mathbb{1}[S+z>0]\, d\Gamma(z) \\
&= \zeta \, \log(1+e^{\frac{S}{\zeta}})
\end{aligned}$$

The expected value of unemployment in NC stage in equation (4) now becomes:

$$\begin{aligned}
\Big(r + \sum_{x'} \rho_u(\boldsymbol{x}'|\boldsymbol{x}) + \chi + \gamma\Big)\, U^g_{NC}(\boldsymbol{x}) = b\,k + \sum_{x'} \rho_u(\boldsymbol{x}'|\boldsymbol{x})\, U^g_{NC}(\boldsymbol{x}') + \chi\, U^g_{PL}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) \\
+ \lambda\, \beta \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \int_{z>-S} \Big(S^g_{NC}(\boldsymbol{x},\boldsymbol{y}) + z\Big) d\Gamma(z)
\end{aligned}$$

where $S^g_{NC}(\boldsymbol{x},\boldsymbol{y})$ denotes the expected surplus and does not depend on z.

Similarly, the expected values from job-to-job transitions are now slightly different in the joint values of a match, and equation (6) becomes:

$$\Big(r + H(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, P^g_{NC}(\boldsymbol{x},\boldsymbol{y}) = (1-\tau)\, f(k,p) + q(\epsilon,\alpha) + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\, \widetilde{P}^g_{NC}(\boldsymbol{x}',\boldsymbol{y})$$
$$+\, \delta^g_{NC}\Big(J(\boldsymbol{y}) + U^g_{NC}(\boldsymbol{x})\Big) + \chi\, \widetilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) + \gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y})$$
$$+\, s\lambda\,\beta \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \int_{z > S - S'} \Big(S^g_{NC}(\boldsymbol{x},\boldsymbol{y}') + z - S^g_{NC}(\boldsymbol{x},\boldsymbol{y})\Big) d\Gamma(z)$$

The balanced flow equations also take into account of the shocks. For example in NC stage, flows in and out of unemployment becomes:

$$u_{NC}(\boldsymbol{x}) \cdot \Big[\chi + \gamma + \lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \cdot \mathrm{Prob}\big[S_{NC}(\boldsymbol{x},\boldsymbol{y}) + z > 0\big] + \sum_{\boldsymbol{x}'} \rho_u(x'|x)\Big]$$
$$= \delta_{NC} \sum_{\boldsymbol{y}} h_{NC}(\boldsymbol{x},\boldsymbol{y}) \cdot \mathrm{Prob}\big[S_{NC}(\boldsymbol{x},\boldsymbol{y}) + z > 0\big]$$
$$+ \sum_{\boldsymbol{x}'} \sum_{\boldsymbol{y}} \rho_e(\boldsymbol{x}|\boldsymbol{x}') h_{NC}(\boldsymbol{x}',\boldsymbol{y}) \cdot \mathrm{Prob}\big[S_{NC}(\boldsymbol{x},\boldsymbol{y}) + z < 0\big]$$
$$+ \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{NC}(\boldsymbol{x}') + \xi_0\,(\boldsymbol{x})$$

where $\mathrm{Prob}\big[S + z > 0\big] = \frac{1}{1+e^{-S/\zeta}}$.

As the scaling parameter $\zeta \to 0$, the model converges to the discrete version. In practice, ζ is set to 0.5 in estimation and it is enough to ensure an equilibrium solution for a wide range of parameter values. Slightly changing the values of ζ does not alter the model moments by much, and thus does not affect parameter estimates.

Appendix I Simulation and estimation

I.1 Simulation

I simulate the model in continuous time and compute moments at annual frequency to match the data structure. Workers' initial human capital is drawn from $\xi_0^g(k)$ for each gender g, and all workers begin life in the *No Child* stage. Initial employment status is assigned according to equilibrium rates: a fraction $emp(\boldsymbol{x})$ of type $\boldsymbol{x}$ workers receive offers drawn from the equilibrium vacancy distribution, where $emp(\boldsymbol{x})$ denotes their equilibrium employment rate in the *NC* stage. The model is then simulated forward for up to 40 years using the monthly Poisson rates specified in Section 3.

I.2 Estimation

I use the following iterative procedure to estimate two sets of parameters, the transition parameters $\boldsymbol{\Lambda} = (\delta^f_{NC}, \delta^f_{YC}, \delta, \vartheta, s_U, s_E)$ and the core parameters $\boldsymbol{\Theta} = (d_1, d_2, K, a, \sigma, \beta, b, \mu^m, \mu^f, M)$.

Step 1: Core moments given transition parameters Given a value for the transition parameters $\boldsymbol{\Lambda}$ obtained from the previous iteration (or an initial guess at the start), I estimate $\boldsymbol{\Theta}$ by minimizing the following quadratic distance

$$L_1(\boldsymbol{\Theta}|\boldsymbol{\Lambda}) = (\hat{m}_1^D - \hat{m}_1^S(\boldsymbol{\Theta}|\boldsymbol{\Lambda}))^T \; \hat{W}_1^{-1} \; (\hat{m}_1^D - \hat{m}_1^S(\boldsymbol{\Theta}|\boldsymbol{\Lambda}))$$

where $\hat{m}_1^D$ is a vector of data moments related to wage profiles of men and women, U-to-E wages and wage growths, proportion of men and women in high- and low-amenity jobs etc. that are described in section 4.3. The vector $\hat{m}_1^S$ are the corresponding model moments from simulations, taking $\boldsymbol{\Lambda}$ as given.

Step 2: Transition moments given core parameters Given the estimate of $\boldsymbol{\Theta}$ obtained from the previous step, I update the estimate of $\boldsymbol{\Lambda}$ by matching appropriate moments related to transitions:

$$L_2(\boldsymbol{\Lambda}|\boldsymbol{\Theta}) = (\hat{m}_2^D - \hat{m}_2^S(\boldsymbol{\Lambda}|\boldsymbol{\Theta}))^T \; \hat{W}_2^{-1} \; (\hat{m}_2^D - \hat{m}_2^S(\boldsymbol{\Lambda}|\boldsymbol{\Theta}))$$

I iterate over these two steps using Monte Carlo Markov Chain (MCMC) until the functions L_1 and L_2 are minimized and the estimates of $\boldsymbol{\Lambda}$ and $\boldsymbol{\Theta}$ converge. The estimation strategy is a good fit for my problem because MCMC is derivative-free, so it is able to handle the non-linearities in the criterion functions due to the discreteness in the model. MCMC can also deal with large parameter spaces and multiple local minima quite well.[45]

I use the sandwich formula to estimate standard errors. Normally, the variance of the converged MCMC chain would provide a direct way to construct valid confidence intervals for the parameter estimates if the optimal weighting matrix is used. But I use a diagonally weighted approach. I will illustrate the computation for the core parameters $\mathbf{\Theta}$ below (the calculation is analogous for the transition parameters $\mathbf{\Lambda}$). The estimated covariance matrix has the form

$$\hat{V}(\hat{\mathbf{\Theta}}) = \left(G'(\hat{\mathbf{\Theta}})\Omega G(\hat{\mathbf{\Theta}})\right)^{-1} G'(\hat{\mathbf{\Theta}})\Omega\, \hat{E}\left[(m_1^S(\hat{\mathbf{\Theta}}) - \hat{m}_1^D)(m_1^S(\hat{\mathbf{\Theta}}) - \hat{m}_1^D)'\right]\Omega G(\hat{\mathbf{\Theta}}) \left(G'(\hat{\mathbf{\Theta}})\Omega G(\hat{\mathbf{\Theta}})\right)^{-1}$$

where Ω is the weight matrix used in the estimation, $G(\hat{\mathbf{\Theta}})$ is the gradient matrix evaluated at the estimated parameters $\hat{\mathbf{\Theta}}$. The variance-covariance matrix of the data moments is computed with 5000 bootstrap samples with replacement.

Estimates for the gradient G are obtained through simulation. Suppose m_1 consists of K moments and $\mathbf{\Theta}$ consists of J parameters. Then the numerical derivatives $\hat{G}(\hat{\mathbf{\Theta}})$ is a $K \times J$ matrix where the j-th column is computed as:

$$\hat{G}_j = \frac{m_1^S(\hat{\mathbf{\Theta}} + h\,\hat{\Theta}_j) - m_1^S(\hat{\mathbf{\Theta}} - h\,\hat{\Theta}_j)}{2\,h\,\hat{\Theta}_j}$$

where m_1^S is the vector of simulated moments evaluated at $\hat{\mathbf{\Theta}} + h\,\hat{\Theta}_j$ and $\hat{\mathbf{\Theta}} - h\,\hat{\Theta}_j$ respectively. The step size of deviation h is a vector of zeros except for one positive element at the j-th position equal to 1%. $\hat{\Theta}_j$ is the j-th element of $\hat{\mathbf{\Theta}}$.

[45]See the discussion in Chernozhukov and Hong (2003) for more details.

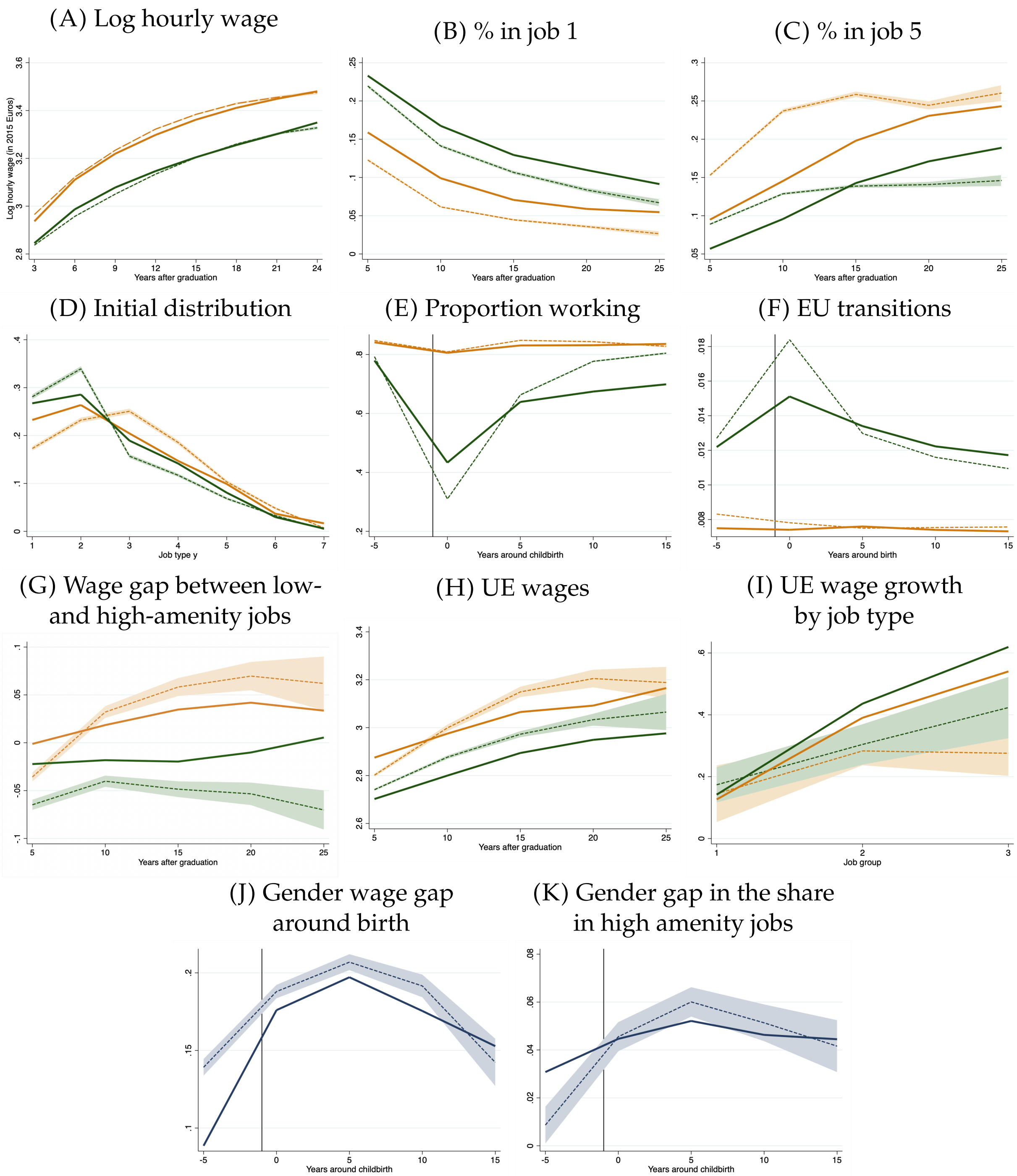
TABLE I1. Summary statistics by job productivity types

Job productivity types	1	2	3	4	5	6	7
Number of workers	27,192	37,155	38,003	41,466	37,309	22,161	13,136
Number of workers per job	2.00	4.20	4.03	4.05	4.04	2.91	2.24
Mean log-wages	2.64	2.96	3.10	3.24	3.39	3.55	3.83
SD of log-wages	0.212	0.043	0.041	0.041	0.044	0.056	0.133
p	1	1.37	1.58	1.82	2.10	2.49	3.28
% **Clerical jobs**	33.51%	7.37%	4.49%	2.91%	1.47%	1.01%	0.70%
% **Associates**	23.03%	18.19%	28.42%	19.54%	13.02%	9.50%	3.46%
% **Professionals**	42.01%	72.26%	63.6%	70.03%	70.89%	59.97%	35.27%
% **Managers**	1.45%	2.17%	3.49%	7.52%	14.62%	29.52%	60.56%

TABLE I2. Calibrated Parameters

Parameters		Estimate	Moments
Fertility rate	χ	0.0085	On average 1.5 children
Ageing rate	γ	0.0042	95% of births happen before age 45
PL ending rate	η^m	0.9524	Parental leave duration of men
	η^f	0.0278	Parental leave duration of women
Production ratio in PL	R	0.7	Ginja, Karimi and Xiao (2023)
Vacancy cost	c	1.6	Lise, Meghir and Robin (2016)
Job distribution	$\varphi(y, \alpha)$	Figure 5	K-means clustering by long-term average wage within firm-occupation

FIGURE I1. Model fit



NOTES: The solid lines represent model moments implied by parameter estimates; the dashed lines are data moments. Green denotes women, and orange denotes men. Shaded areas correspond to bootstrap standard errors from 5000 random draws of the data.

Appendix J Model with no capacity constraint

With no capacity constraints, vacancy values are set to zero for all jobs. This reflects free entry that drives profits to zero when firms are allowed to post an unconstrained number of vacancies. In this environment, the surplus function for any given worker type increases monotonically with job productivity and cannot exhibit the hump-shaped pattern seen in Figure 6. As a result, the model does not generate sorting along the productivity dimension. Highly skilled workers do not disproportionately match with highly productive jobs, even if the CES production function were to exhibit strong complementarity.

I estimate this alternative model with the same targeted data moments. Since job allocations are no longer endogenous in this model, I calibrate the transition parameters Λ and estimate only the core parameters Θ.

As reported in Table J1, the production function estimates imply that worker skills and job productivity are weak complements ($\sigma = 0.54$) in the alternative model, as opposed to strong complements ($\sigma = -14.49$) in the baseline model. In order to match the increasing wages over the life-cycle without any sorting mechanism, the alternative model attributes the observed wage growth in the data to a higher rate of human capital accumulation (baseline HC growth rate $d_1 = 0.023$ in the alternative model compared with 0.001 in the baseline model).

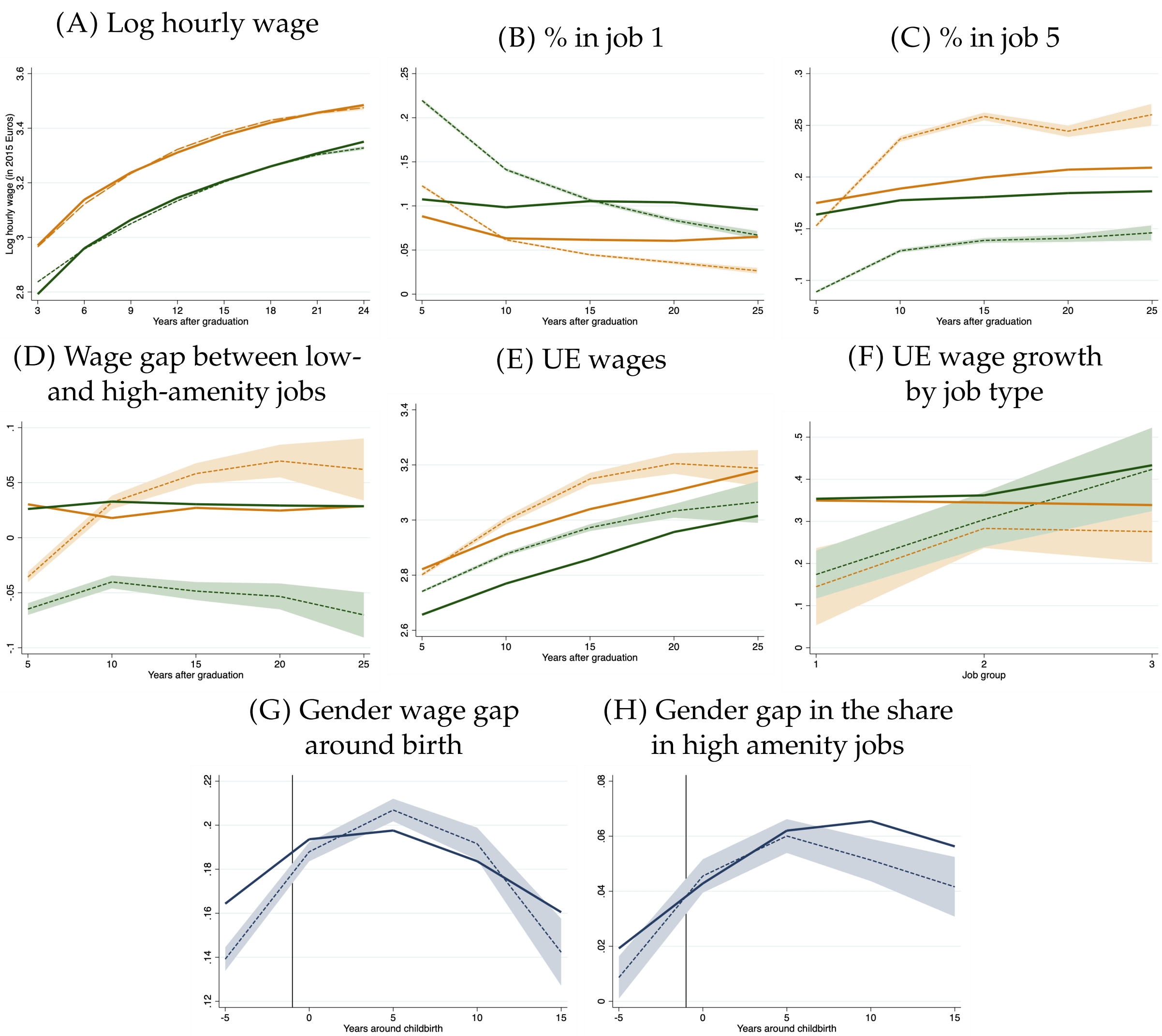
Unsurprisingly, the alternative model fails to replicate the observed sorting patterns of men and women over the life-cycle, although it manages to fit the wage profiles. In the data, the shares of men and women in low-productivity jobs (category 1) decline with experience, while the shares in high-productivity jobs (category 5) rise substantially (Figure J1, panels B and C). In contrast, the model generates nearly flat profiles for both groups, along with a much smaller gender gap in high-productivity positions.

TABLE J1. Parameter estimates of model with no capacity constraint (NCC)

$\boldsymbol{\Theta}$ parameters		NCC Estimates (S.E.)	Baseline Estimates (S.E.)
Complementarity	σ	0.540	−14.491
		(0.384)	(1.167)
Relative productivity	a	0.855	0.848
		(0.068)	(0.021)
TFP	A	13.026	29.526
		(4.617)	(1.392)
Baseline HC rate	d_1	0.023	0.001
		(0.002)	(0.002)
Proportional HC rate	d_2	0.000	0.010
		(0.015)	(0.003)
Men's value for amenities	μ^m	0.163	0.767
		(0.075)	(0.084)
Women's value for amenities	μ^f	0.174	0.883
		(0.082)	(0.092)
Preference increase in motherhood	M	0.425	1.815
		(0.580)	(1.923)
Worker's bargaining	β	0.665	0.519
		(0.192)	(0.016)
Home productivity	b	5.437	5.109
		(1.791)	(0.897)
Initial distribution - men	ξ^m	2.070	2.201
		(1.079)	(0.936)
Initial distribution - women	ξ^f	2.650	2.798
		(1.286)	(1.013)

NOTES: Standard errors are computed with 5000 bootstrap samples as described in Appendix I. Baseline estimates are from Table 1.

FIGURE J1. No capacity constraint model fit



NOTES: The solid lines represent model moments implied by parameter estimates; the dashed lines are data moments. Green denotes women, and orange denotes men. Shaded areas correspond to bootstrap standard errors from 5000 random draws of the data.

Appendix K Model with endogenous fertility

In the baseline model, all workers transition to a stage with young child following an exogenous fertility shock χ. In this section, I develop and estimate an alternative model with endogenous fertility.

At rate χ^g, workers of gender g receive an opportunity to have a child and choose whether to do so by comparing the continuation value of entering the *Parental Leave/Young Child* stage with the value of remaining in the *No Child* stage:

$$\begin{aligned}
&\Big(r + H_e(k,p) + \delta^g_{NC} + \chi^g + \gamma\Big)\, P^g_{NC}(\boldsymbol{x},\boldsymbol{y}) = \\
&\underbrace{f(k,p)}_{\text{flow output}} + \underbrace{q(\epsilon,\alpha)}_{\text{value for amenities}} + \underbrace{\delta^g_{NC}\Big(J(\boldsymbol{y}) + U^g_{NC}(\boldsymbol{x})\Big)}_{\text{exogenous separation}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\, \widetilde{P}^g_{NC}(\boldsymbol{x}',\boldsymbol{y})}_{\text{HC accumulation}} \\
&+ \underbrace{\chi^g \max\Big\{\tilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}), P^g_{NC}(\boldsymbol{x},\boldsymbol{y})\Big\}}_{\text{fertility choice}} \\
&+ \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\ \beta\Big[\underbrace{S^g_{NC}(\boldsymbol{x},\boldsymbol{y}')}_{\text{poaching job surplus}} - \underbrace{S^g_{NC}(\boldsymbol{x},\boldsymbol{y})}_{\text{current job surplus}}\Big]^+ + \underbrace{\gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y})}_{\text{ageing}}
\end{aligned} \tag{29}$$

In *Parental Leave* and *Young Child* stages, the worker derives an additional flow utility κ from spending time with children.

$$\begin{aligned}
(r + \delta^g_{YC} + \eta^g + \gamma)\, P^g_{PL}(\boldsymbol{x},\boldsymbol{y}) = &\underbrace{R\, f(k,p)}_{\text{reduced flow output}} + \underbrace{\kappa}_{\text{value for children}} + \underbrace{\delta^g_{YC}\Big(J(\boldsymbol{y}) + U^g_{PL}(\boldsymbol{x})}_{\text{exogenous separations}}\Big) \\
&+ \underbrace{\eta^g\, \widetilde{P}^g_{YC}(\boldsymbol{x},\boldsymbol{y})}_{\text{PL ends}} + \underbrace{\gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y})}_{\text{ageing}}
\end{aligned} \tag{30}$$

$$\begin{aligned}
(r + H_e(k,p) + \delta^g_{YC} + \chi^g + \gamma)\, P^g_{YC}(\boldsymbol{x},\boldsymbol{y}) = &\ f(k,p) + \kappa + q^g_{YC}(\epsilon,\alpha) + \sum_{k'} \rho_e(k'|k,p)\, \widetilde{P}^g_{YC}(\boldsymbol{x}',\boldsymbol{y}) \\
&+ \delta^g_{YC}\Big(J(\boldsymbol{y}) + U^g_{YC}(\boldsymbol{x})\Big) + \chi^g\, \widetilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) + \gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y}) \\
&+ \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\ \beta\, [S^g_{YC}(\boldsymbol{x},\boldsymbol{y}') - S^g_{YC}(\boldsymbol{x},\boldsymbol{y})]^+
\end{aligned} \tag{31}$$

I assume the child has left the household in the *Non-Fecund* stage, so the worker no longer enjoys flow utility κ. P^g_{NF} is therefore identical to equation (20).

Non-employment values are modified analogously. In the *NC* stage, unemployed workers also choose whether to have a child by comparing $U^g_{PL}(\boldsymbol{x})$ with $U^g_{NC}(\boldsymbol{x})$. The fertility term in U^g_{NC} is therefore replaced by $\chi^g \max\left\{U^g_{PL}(\boldsymbol{x}), U^g_{NC}(\boldsymbol{x})\right\}$. U^g_{PL} and U^g_{YC} add κ to the flow utility as in equations (30)–(31). U^g_{NF} is unchanged.

The balanced flow equations take the same form as equations (21)–(28), augmented to reflect endogenous fertility decisions. Specifically, the fertility outflow term χ in equations (21) and (25) is replaced by $\chi^g \cdot \mathbb{1}\left[\tilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) > P^g_{NC}(\boldsymbol{x},\boldsymbol{y})\right]$ for employed workers and $\chi^g \cdot \mathbb{1}\left[U^g_{PL}(\boldsymbol{x}) > U^g_{NC}(\boldsymbol{x})\right]$ for non-employed workers. Flows in all other stages are unchanged.

The wage equations (13)–(15) are modified analogously, with the χ term in each multiplied by $\mathbb{1}\left[\tilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) > P^g_{NC}(\boldsymbol{x},\boldsymbol{y})\right]$, reflecting that wages adjust toward the *PL/YC* schedule only when the worker actually chooses to have a child.[46]

This specification estimates three fertility-related parameters in addition to $\boldsymbol{\Theta}$ and $\boldsymbol{\Lambda}$: gender-specific fertility arrival rates χ^m and χ^f, and the flow value of children κ. The parameters χ^m and χ^f determine aggregate fertility levels for men and women, while κ sets a threshold for the change in expected match values below which workers may opt not to have children. Fertility rates by initial job category are therefore informative target moments for identifying these parameters.

A worker's initial job is defined as the category in which they spend the most time during the first three years of potential experience, prior to childbirth. Fertility is measured by the number of children by age 50. I focus on initial jobs because later job outcomes are more endogenous to early career choices and may already be affected by fertility decisions.

The parameter estimates are reported in Table K1, and most remain close to those in the baseline model. The estimated fertility arrival rate $\hat{\chi}^f = 0.0092$ exceeds the baseline estimate of 0.0085. Under endogenous fertility, some women who draw high productivity jobs early in their careers choose not to have children, so the model requires a higher unconditional arrival rate to match average completed fertility in the data.

As shown in Figure K1, the model matches most targeted moments well. However, endogenizing fertility causes the model to severely underpredict women's fertility in high-productivity job categories. This prediction is not supported in the data. Instead, fertility is slightly higher among men and women with better initial jobs, consistent with the high monetary costs of children making higher-income households more likely to have children — a mechanism the model abstracts away from.

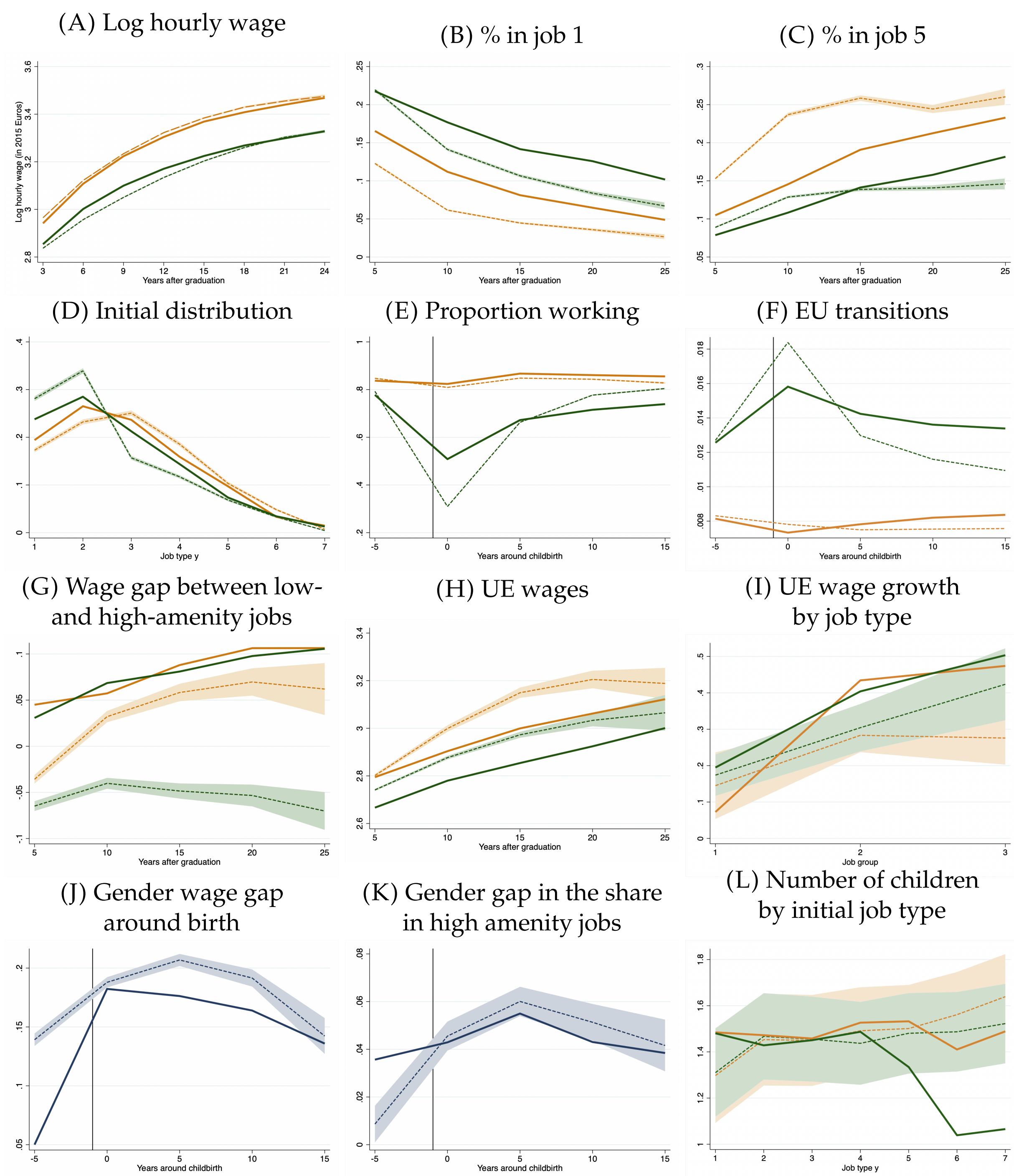
[46]Since parents enjoy the same flow utility of children whether employed or not, κ does not enter the wage equations in the *PL/YC* stages.

TABLE K1. Parameter estimates of model with endogenous fertility

Parameters		Endog. fertility Estimates	Baseline Estimates
Complementarity	σ	-11.162	-14.491
		(0.646)	(1.167)
Relative productivity	a	0.809	0.848
		(0.058)	(0.021)
TFP	A	23.240	29.526
		(4.509)	(1.392)
Baseline HC rate	d_1	0.001	0.001
		(0.001)	(0.002)
Proportional HC rate	d_2	0.007	0.010
		(0.004)	(0.003)
Men's value for amenities	μ^m	0.799	0.767
		(0.062)	(0.084)
Women's value for amenities	μ^f	0.706	0.883
		(0.094)	(0.092)
Preference increase in motherhood	M	1.542	1.815
		(1.379)	(1.923)
Worker's bargaining	β	0.583	0.519
		(0.024)	(0.016)
Home productivity	b	6.297	5.109
		(1.079)	(0.897)
Initial distribution - men	ξ^m	2.105	2.201
		(0.920)	(0.936)
Initial distribution - women	ξ^f	2.521	2.798
		(1.112)	(1.013)
Fertility rate - men	χ^m	0.0086	0.0085
		(0.001)	
Fertility rate - women	χ^f	0.0092	0.0085
		(0.001)	
Flow value of children	κ	5.018	
		(1.944)	
Women's separation rate in NC	δ_{NC}	0.011	0.011
		(0.001)	(0.001)
Women's separation rate in YC	δ_{YC}	0.018	0.015
		(0.003)	(0.003)
Men's separation rate	δ	0.006	0.008
		(0.002)	(0.002)
Matching efficiency	ϑ	0.107	0.110
		(0.009)	(0.007)
Relative search intensity in unemployment	s_U	0.670	0.687
		(0.297)	(0.405)
Relative search intensity in employment	s_E	0.516	0.526
		(0.245)	(0.214)

NOTES: Standard errors in parentheses. Baseline estimates are from Table 1. Fertility rate χ is not gender-specific in the baseline model.

FIGURE K1. Endogenous fertility model fit



NOTES: The solid lines represent model moments implied by parameter estimates; the dashed lines are data moments. Green denotes women, and orange denotes men. Shaded areas correspond to bootstrap standard errors from 5000 random draws of the data.

Appendix L Counterfactuals

TABLE L1. Counterfactual gender gaps in log hourly wages corresponding to Figure 8

	Years since graduation							
	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24
Baseline gap (log points)	**9.07**	**12.26**	**14.02**	**14.94**	**15.54**	**15.38**	**14.73**	**13.24**
Resulting wage gaps:								
(1) Equal parental leave	7.95	9.18	10.37	10.83	11.06	10.87	9.93	8.76
(2) + Equal separations	7.32	6.77	5.97	5.14	5.12	4.45	3.53	2.54
(3) + Equilibrium allocations	6.98	6.10	5.18	4.38	4.37	3.85	3.11	2.07
(4) + Equilibrium wages	3.31	3.35	3.57	3.54	4.02	3.80	3.24	2.23
(5) + Same preferences	3.41	2.54	2.21	2.03	2.42	2.29	1.80	1.03
Marginal changes:								
(1) - baseline	-1.12	-3.08	-3.64	-4.11	-4.48	-4.51	-4.80	-4.48
(2) - (1)	-0.63	-2.41	-4.40	-5.68	-5.94	-6.42	-6.40	-6.22
(3) - (2)	-0.35	-0.67	-0.80	-0.77	-0.75	-0.61	-0.42	-0.47
(4) - (3)	-3.66	-2.75	-1.60	-0.84	-0.35	-0.05	0.13	0.16
(5) - (4)	0.10	-0.82	-1.37	-1.51	-1.59	-1.51	-1.44	-1.20

TABLE L2. Effect of each channel on gender wage gap and their interactions (log points)

	Years since graduation							
	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24
Baseline gap (log points)	**9.07**	**12.26**	**14.02**	**14.94**	**15.54**	**15.38**	**14.73**	**13.24**
A. Individual effect of channel:								
(1) Equal parental leave	-1.12	-3.08	-3.64	-4.11	-4.47	-4.51	-4.80	-4.48
(2) Equal separations	-0.71	-2.54	-3.99	-5.05	-5.73	-6.06	-5.94	-5.91
(3) Equilibrium allocations	-0.51	-0.92	-0.97	-0.78	-0.62	-0.50	-0.56	-0.55
(4) Equilibrium wages	-4.17	-3.71	-2.75	-1.91	-1.23	-0.85	-0.70	-0.72
(5) Same preferences	0.03	-0.64	-1.05	-1.34	-1.45	-1.62	-1.55	-1.43
B. Interactions:								
Add up (1) and (2)	-1.83	-5.62	-7.64	-9.16	-10.21	-10.57	-10.74	-10.39
Joint (1) and (2)	**-1.74**	**-5.49**	**-8.04**	**-9.79**	**-10.42**	**-10.93**	**-11.20**	**-10.70**
Add up (1) to (3)	-2.34	-6.54	-8.61	-9.93	-10.82	-11.06	-11.30	-10.95
Joint (1) to (3)	**-2.09**	**-6.16**	**-8.84**	**-10.56**	**-11.17**	**-11.53**	**-11.62**	**-11.17**
Add up (1) to (4)	-6.51	-10.25	-11.36	-11.84	-12.05	-11.92	-12.00	-11.67
Joint (1) to (4)	**-5.76**	**-8.91**	**-10.44**	**-11.40**	**-11.52**	**-11.58**	**-11.49**	**-11.01**
Add up (1) to (5)	-6.47	-10.89	-12.41	-13.18	-13.50	-13.54	-13.55	-13.11
Joint (1) to (5)	**-5.66**	**-9.72**	**-11.81**	**-12.91**	**-13.11**	**-13.09**	**-12.93**	**-12.21**

TABLE L3. Counterfactual female shares in top jobs corresponding to Figure 9

	Years since first childbirth								
	-9 to -7	-6 to -4	-3 to -1	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18
Baseline female share (%)	**33.0**	**34.8**	**37.4**	**33.1**	**34.4**	**35.2**	**35.1**	**36.8**	**36.3**
Female shares after:									
(1) Equal parental leave	33.0	34.8	37.4	37.9	40.1	40.0	39.5	40.7	39.9
(2) + Equal separations	36.6	37.7	38.7	40.9	43.3	44.3	45.3	46.5	45.8
(3) + Equilibrium allocations	41.3	43.6	43.2	46.2	46.8	46.5	47.7	48.7	47.6
(4) + Same preferences	42.9	43.5	41.8	45.0	45.8	45.8	47.0	47.9	47.7
Marginal changes:									
(1) - baseline	0.0	0.0	0.0	4.8	5.7	4.8	4.4	3.8	3.6
(2) - (1)	3.6	2.9	1.4	3.1	3.2	4.3	5.7	5.8	5.9
(3) - (2)	4.7	5.9	4.4	5.3	3.4	2.2	2.5	2.2	1.8
(4) - (3)	1.6	0.0	-1.4	-1.3	-1.0	-0.7	-0.7	-0.7	0.1

TABLE L4. Effect of each channel on female shares in top jobs and their interactions (%)

	Years since first childbirth								
	-9 to -7	-6 to -4	-3 to -1	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18
Baseline female share (%)	**33.0**	**34.8**	**37.4**	**33.1**	**34.4**	**35.2**	**35.1**	**36.8**	**36.3**
A. Indiv. effect of channel:									
(1) Equal parental leave	0.0	0.0	0.0	4.8	5.7	4.8	4.4	3.8	3.6
(2) Equal separations	3.6	2.9	1.4	3.4	3.9	4.2	4.5	4.1	4.5
(3) Equilibrium allocations	4.5	4.6	3.8	4.4	4.2	2.9	3.2	2.2	1.6
(4) Same preferences	-2.5	-1.4	-0.6	-0.2	-0.4	-0.2	-0.2	0.0	0.0
B. Interactions:									
Add up (1) to (3)	8.1	7.4	5.1	12.5	13.8	11.9	12.1	10.1	9.7
Joint (1) to (3)	**8.4**	**8.7**	**5.8**	**13.1**	**12.4**	**11.3**	**12.6**	**11.8**	**11.3**
Add up (1) to (4)	5.7	6.0	4.6	12.4	13.4	11.7	11.9	10.1	9.7
Joint (1) to (4)	**10.0**	**8.7**	**4.4**	**11.9**	**11.4**	**10.6**	**11.9**	**11.1**	**11.4**