Equilibrium Sorting and the Gender Wage Gap

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November 28, 2024

Abstract

Women face a "glass ceiling" in career advancement, with considerable underrepresentation in high-paying, high-status positions. This may arise because employers anticipating more career interruptions from women might allocate them to low-productivity jobs where turnover is less costly. This paper develops an equilibrium search model to quantify the extent to which the "glass ceiling" could be attributed to employers' job allocations decisions on one hand, and women's experience accumulation and preference for family-friendly amenities on the other hand. Estimating the model on administrative employer-employee data from Finland, I find that employers' differential wage-setting and job allocations account for a substantial proportion of the gender wage gap in early career, whereas gender differences in labor force attachment explain most of the wage gap in late career. The model reveals that sharing parental leave more equally between men and women closes gender gaps both before and after having children.

JEL-codes: J23, J64, J24, J16, E24, J32
Keywords: Gender wage gap, sorting, equilibrium search model, human capital, child penalty, non-wage amenities

*Department of Economics, Duke University. Email: pengpeng.xiao@duke.edu. I am grateful to my advisors Costas Meghir, Ilse Lindenlaub and Joseph Altonji for their continued guidance and support. I thank Chris Flinn, Peter Arcidiacono, Tomi Kyyrä, Cormac O'Dea, Roope Uusitalo and Christian Moser for helpful suggestions. I also thank seminar participants at VATT, Minnesota, CEMFI, IIES, Warwick, IFS, Tilburg, Arizona State, UNC-Chapel Hill, Duke, UVA, Brown, Notre Dame, NUS, Insper, Vanderbilt, Nebraska, Carnegie Mellon, Penn State, NYU, Bank of Portugal, Toulouse, Chicago (Lifecycle Working Group), UW-Madison, FGV, WU-St. Louis, CUHK, Copenhagen Business School, Wharton, Stony Brook, Johns Hopkins, as well as attendants at SOFI Gender Webinar, 2021 Essex/RHUL/Bristol Junior SaM Workshop, 2021 SOLE Meetings, 2021 Barcelona GSE Summer Forum (Income Dynamics and the Family), 2021 SITE Workshops (Micro and Macro of Labor Markets), 2021 IZA Workshops (Labor Market Institutions), 2022 Dale T. Mortensen Workshop, 2023 Junior Micro/Macro Labor Conference (Columbia Business School), and 2023 Workshop on Families, Human Capital and Inequality (Mannheim and Bonn). I would especially like to thank Kristiina Huttunen and VATT Institute for Economic Research for facilitating access to the data. I acknowledge funding from Cowles Foundation and MacMillan Fellowship. All mistakes are my own.

1 Introduction

The gender wage gap expands substantially over the life-cycle, as women advance much more slowly in their careers than their male counterparts. Women remain heavily under-represented in high-earning, high-status occupations, and a "glass ceiling" is documented in many countries.[1] A large proportion of the gender wage gap in the 21st century can be attributed to men and women sorting into different industries, occupations and firms.[2] However, less is known about the extent to which this differential sorting is driven by *workers' preferences* over job attributes or *employers' decisions* to assign men and women to different positions. Since match formation and wages are influenced by both workers and firms in the labor market, it is important to consider both labor supply and demand sides when designing policies aimed at reducing gender inequality. On one hand, we have policies to bolster women's labor supply, their stable employment after childbirth and access to top-level jobs; but on the other hand, the same policies could have unintended consequences when employers' responses are taken into account.

Using administrative matched employer-employee data from Finland, I first document gender gaps in labor market outcomes for university graduates. Over the life-cycle, the unconditional gap between men and women's mean hourly wages increases rapidly from 12 log points at labor market entry to 20 log points after ten years. A "glass ceiling" in women's career starts to emerge six years after labor market entry, as the most successful women fail to move into high-paid managerial positions. Female and male university graduates have similar employment rates before having children, but women take 18 months of parental leave for each child while men take about 2 months on average. After parental leave, women's employment remains precarious as they are more than twice as likely as men to transition from employment to non-employment. Compared to men, women are also more likely to reduce hours, switch to part-time jobs, and move to jobs with better amenities (in terms of a family-friendly index) after childbirth.

Motivated by the empirical observations, this paper examines three common explanations for the diverging career paths of men and women over the life-cycle. First, work interruptions after having children hinder women's experience accumulation on the job, so they progress more slowly than their male colleagues.[3] Second, women may sort into

[1]For example in Sweden (Albrecht, Bjorklund and Vroman, 2003), the US (Matsa and Miller, 2011), Norway (Bertrand, Black, Jensen and Lleras-Muney, 2018), and many developing countries (Pande and Ford, 2012).

[2]See Blau and Kahn (2017) and Altonji and Blank (1999) for comprehensive reviews on the explanations of the gender wage gap.

[3]For an overview of "motherhood penalties" across countries, see Kleven, Landais, Posch, Steinhauer

jobs that pay lower wages but offer more flexibility and other family-friendly amenities.[4] Third, employers might anticipate women to have more fertility-related separations and absence that are costly for the firm, so they allocate different jobs to men and women (e.g. placing women on "mommy tracks"), or offer different wages to male and female workers for the same job, or both.[5]

In order to quantify the relative importance of the above channels, I develop an equilibrium search and matching model that combines human capital accumulation, preferences, and employer-side mechanisms. In the model, workers gain skills and conduct on-the-job search while employed, but human capital stagnates while unemployed or on parental leave. Male and female workers compete for the same jobs in the same labor market, but can differ in turnover rates, childcare responsibilities, and preferences for family-friendly amenities. Frictions in the labor market means that it takes time for workers to find jobs and for vacancies to find workers.[6]

To investigate sorting, the model incorporates a fundamental scarcity of jobs similar to Shimer and Smith (2000). This scarcity creates a unique dynamic where employers must carefully weigh the trade-offs between hiring men and women, considering the option values of keeping the vacancy open and waiting for potentially better matches. Hiring a woman may result in a lower match surplus due to the costs associated with higher expected turnovers. Therefore, differential sorting by gender across occupations and firms is not only driven by workers' preferences, but also production complementarities and employers' cost concerns.

In a capacity-constrained environment, highly productive jobs (such as managerial positions) may set higher qualification thresholds for employing women than men. This occurs because vacancy values are positive and increasing along the job productivity ladder, and top positions have more to lose when a worker departs. Since top managerial jobs have very high outside option values, the expected future production values with women may not be sufficiently large to make the match worthwhile. This leads to the emergence

and Zweimüller (2019) and Kleven, Landais and Leite-Mariante (2024).

[4] Felfe (2012), Goldin (2014) and Wiswall and Zafar (2017) show that women sort into occupations with temporal flexibility and fewer working hours. Adda, Dustmann and Stevens (2017) and Hotz, Johansson and Karimi (2018) find long-term career consequences for women working in family-friendly occupations.

[5] Although employers' expectations might be correct on average and their decisions are rational, such differential wage-setting and job allocation practices towards men and women would constitute the notion of statistical discrimination as in Arrow (1972) and Phelps (1972) – employers cannot observe the individual's labor force attachment, so they make decisions for each individual worker based on his/her group average characteristics.

[6] This paper remains agnostic about the exact source of frictional costs. Conceptually, the cost of a vacancy in this paper might include the cost of searching, recruiting or training new workers, reorganizing the workforce, paying overtime hours, etc. in addition to the opportunity cost of forgone production.

of a “glass ceiling” effect, where high-end jobs have a lower probability of matching with women than men of the same productivity. I show that a conventional job ladder model without capacity constraints cannot generate the “glass ceiling” phenomenon evident in empirical observations.

The model not only allows me to quantify the mechanisms underlying the large “motherhood penalties” after childbirth, but also helps to unpack gender gaps that already exist before childbirth. By combining three common explanations into one unified framework, I also use the model to examine any interactions between the channels and assess the effectiveness of potential policy interventions.

Using the method of simulated moments, I estimate the model on the Finnish matched employer-employee data combined with Quality of Work Life surveys on job amenities. The model accurately fits key moments of the data. The CES production function estimates show strong complementarity between workers’ skills and job productivity, leading to positive assortative matching (PAM) for both men and women. The estimates for human capital accumulation rates are positive and increasing in job productivity. This implies that even though men and women face the same learning rates for a given job, their mean human capital can still diverge over the life-cycle as they sort into different jobs and work for different amounts of time. Prior to parenthood, estimates for male and female preferences are similar; however, women’s valuation of family-friendly amenities increases substantially following childbirth.

Given the parameter estimates, I first decompose the life-cycle gender wage gap. The model reveals that 4 log points (out of 9) of the gender wage gap during the initial stages of a career can be attributed to employers’ differential sorting and wage-setting by gender based on fertility-related concerns. As workers progress beyond child-rearing ages, the influence of employers’ gender-specific decisions diminishes. Instead, a significant majority (11 out of 15 log points) of the wage gap in later career stages can be ascribed to gender disparities in labor force attachment and the cumulative deficit in women’s human capital. Amenity preferences affect sorting patterns primarily after childbirth and is responsible for about 1.5 log points of the overall wage gap over the life-cycle. The residual wage gap, which could be due to employers’ taste-based discrimination or initial productivity differences between men and women, accounts for approximately 17% of the total gap.

I also use the model to decompose the factors underlying women’s underrepresentation in top positions. In the baseline, women face fewer opportunities than men at high-productivity jobs before having children, but there is no gender difference in job allocations in the non-fertile periods. If employers were to anticipate the same parental leave and separation rates across genders, women would gain access to more high-end jobs and reject more low-end jobs in early career, and the share of women in top positions would increase from 38 to 43 percent in the six years prior to childbirth. Since high-productivity jobs provide more learning opportunities, the human capital channel interacts with the sorting channel and amplifies the effects of job access. Enhanced skill levels make women less likely to endogenously quit their jobs after having children, so the equilibrium sorting channel has a sustained positive effect of 4 to 2 percentage points in the female share five to fifteen years post-childbirth.

The model also provides insights into the efficiency-equity trade-offs of potential policies. I simulate the effects of three policy interventions. First, a "daddy months" expansion that shifts two months of parental leave from women to men closes the wage gap by 13% throughout the life-cycle. It incurs a small welfare loss of 0.02% because men are already at more advanced positions than women by the time they have children, so the production loss from men staying home for two months is less than compensated by women working two months more. Second, an equal hiring policy in top jobs improves women's representation in managerial positions, but employers undo this policy by exerting more wage discounts. Moreover, the increased access to top jobs is short-lived as the policy does not address the negative impacts of motherhood. However, the human capital gains in high-quality jobs have positive externalities, which increase social welfare by 0.01%. Third, an equal pay counterfactual shows that requiring firms to pay the same wage to similar men and women closes the gender wage gap by 15% on average. However, this requirement has unintended consequences as employers adjust on the job allocation margin. Women are more likely to be unemployed, and the proportion of women in top jobs decreases by 1 percentage point twenty years after labor market entry.

Overall, the results suggest that achieving gender equality at the workplace would be difficult without greater equality in family responsibilities, such as sharing parental leave more equally between men and women. The challenge arises from two key factors: first, the strong forces of positive assortative matching in the high-skill labor market mean that women would gravitate towards lower-tier jobs if they accumulate less experience; and second, the substantial equilibrium responses of labor demand imply that employers would continue to suppress women's wages and job positions if they anticipate women to be the main caretaker.

The results in this paper contribute both theoretically and empirically to the literature studying gender differences in job search and wage outcomes. First and foremost, the paper adds to the literature that employs search models to examine the gender pay gap (Bowlus, 1997; Flabbi, 2010; Bartolucci, 2013; Flabbi and Mabli, 2018; Gray, 2021; Morchio and Moser, 2024; Amano, Baron and Xiao, 2021; Flinn, Todd and Zhang, 2024). Beyond studying gender gaps in wages, the model in this paper introduces capacity constraints in job slots, requiring employers to make decisions about both wage offers and job allocations. Without the job-allocations margin, conventional search models typically attribute the gender segregation across jobs to other factors, e.g. exogenous job productivity distributions that are different for men and women (Bartolucci, 2013; Amano, Baron and Xiao, 2021), or gender-specific recruiting costs (Morchio and Moser, 2024). However, the parameters governing gender segregation in these studies are invariant to policies, limiting the scope to analyze the impact of interventions like family leave or equal pay regulations.

In contrast, this paper allows policies to influence gender disparities in *job opportunities* by endogenizing the sorting margin, providing a more comprehensive examination of the equilibrium effects. Incorporating both wage and sorting margins is crucial within a life-cycle dynamic framework, because limited opportunities early in one's career can significantly impact human capital accumulation and subsequent job choices. These early career constraints may lead to long-term wage consequences, offering a more nuanced understanding of the gender pay gap over the life-cycle.

The model in this paper is built on a body of search-matching literature with wage bargaining (Dey and Flinn, 2005; Cahuc, Postel-Vinay and Robin, 2006), human capital accumulation (Herkenhoff, Lise, Menzio and Phillips, 2024; Lise and Postel-Vinay, 2020), and job amenities (Dey and Flinn, 2008; Taber and Vejlin, 2020). Importantly, sorting of workers to jobs depends on match complementarities in the production function,[7] and I estimate a flexible CES production function rather than a multiplicative or Cobb-Douglas function commonly used in the search literature. By integrating several key modeling components into a single framework, my paper not only quantifies the various factors contributing to gender gaps, but also explores the complex interactions that individual studies may overlook.

Finally, the paper is related to the theoretical literature that links women's child-related career interruptions to firms' statistical discrimination. There are many ways to model firms' decisions in the presence of frictions (e.g. information asymmetry, search costs, training costs etc.).[8] One major advantage of adopting a frictional job search frame-

[7]See for example two-sided matching market (Becker, 1973; Shimer and Smith, 2000; Eeckhout and Kircher, 2011), with on-the-job search (Lise, Meghir and Robin, 2016; Hagedorn, Law and Manovskii, 2017; Lopes de Melo, 2018; Borovickova and Shimer, 2024), and with endogenous search effort (Bagger and Lentz, 2018).

[8]Albanesi and Olivetti (2009), Gayle and Golan (2012) and Thomas (2024) assume asymmetric informa-

work in my paper is tractability. It allows the model to accommodate granular job types (occupations within firms) and many worker types, which gives rise to a clear notion of career paths or "job ladders" over the life-cycle. The rich heterogeneity in job types allows me to analyze gender disparities in career advancement at low vs. high ends of the job productivity distribution, and quantify any welfare losses resulting from worker-job mismatches at different stages of the career.

2 Data

In this section, I will briefly describe the datasets and show a number of empirical patterns related to gender differences in the labor market.

2.1 Datasets

The Finnish Longitudinal Employer-Employee Data (FOLK) provides information on workers' demographics, monthly employment histories, children's birth dates and parental leave claims for the full population between years 1988 and 2016. Parental leave duration is inferred from the annual parental leave allowance and home care allowance claims according to a schedule detailed in Appendix C. The Structure of Earnings Statistics (SES) 1995-2013 contains full-time equivalent hourly wages, part-time status, contracted hours and 4-digit occupation codes. Such detailed occupation codes are typically not available from tax registers.[9]

Since educated workers experience the largest increase in the gender wage gap over the life-cycle, in this paper I will focus on individuals who obtained master's degrees[10] in the years 1988 to 2005 so that we observe at least 8 years of labor market activities. Appendix B provides more details on sample restrictions.

tion about worker's labor market attachment, and employers' priors about women's family responsibilities lead to statistically discrimination. The statistical discrimination model in Barron, Black and Loewenstein (1993) assumes exogenous gender differences in quit rates (with symmetric information), and training costs lead firms to sort women into jobs that require less training.

[9] The SES covers 55 to 75 percent of private sector workers depending on the year, and under-samples small firms. Since I do not include small firms with 2 workers or less, data coverage is not a big issue. In the estimation, I use sample weights to account for potential missing data from small firms.

[10] Master's degree in Finland is roughly equivalent to US bachelor's degree, since Finnish students who get into academic-track bachelor's programs are automatically enrolled in the master's programs.

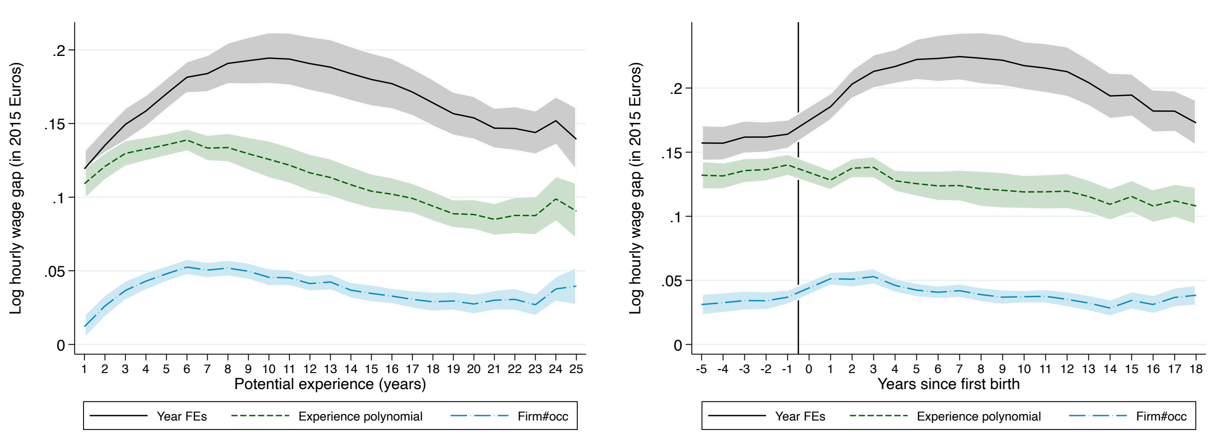
2.2 Descriptive decomposition of the gender wage gap

Women have overtaken men in educational attainment in Finland. However, women's labor market outcomes do not seem to catch up to those of their male classmates.

To investigate the evolution of the gender wage gap over the life-cycle, I first decompose it descriptively by successively adding more controls. Figure 1(A) shows the difference between men and women's log hourly wages by years since graduation (potential experience): *(i)* unadjusted (only with year fixed effects); *(ii)* adjusted for a quadratic in actual experience[11] in addition to *(i)*; and *(iii)* adjusted for a full set of interactions between 4-digit occupation and firm dummies in addition to *(i)* and *(ii)*. Figure 1(B) does the same decomposition exercise in the years around the first childbirth.

FIGURE 1. Descriptive decomposition of the gender wage gap

(A) Over the life-cycle (B) Around childbirth



NOTES: The lines represent the coefficients on the male dummy interacted with potential experience or years since childbirth. Shaded areas represent 95% confidence intervals. Standard errors are clustered at the firm level. The coefficients are obtained from regressions of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience in addition to (i); (iii) a full set of interactions of firm and occupation dummies in addition to (i) and (ii).

The unadjusted gender wage gap in Figure 1(A) increases from 12 log points at labor market entry to 20 log points in 10 years, and then declines slowly to 15 log points in 25 years (when workers are above age 50). Since women spend more time in non-employment especially after childbirth, actual experience explains more and more of the wage gap between men and women over the course of their careers.

[11] Actual experience is defined as the cumulative number of months a person has worked after university graduation. Since both men and women might work before obtaining master's degrees, I calculate their formal labor market experience after bachelors' graduation, excluding short-term employment of 3 months or less and excluding summer internships. By the time they graduate with master's degree, men have 1.9 years of actual experience while women have 1.6 years. The difference is not statistically significant.

There is still an “unexplained” gap (the bottom blue line in Figure 1(A)) of about 4 log points after adding 4-digit occupation fixed effects, firm fixed effects and their interactions, suggesting that men and women are offered different wages even when they have the same actual experience and work in the same job within the same firm. Appendix Figure A1 shows similar results after adding additional controls for the field of study in university.[12] The existence of “unexplained” gaps might imply unequal pay for similarly qualified workers, potentially due to statistical or taste-based discrimination.

To investigate the impact of children, a similar descriptive decomposition is conducted for the years around the birth of the first child. Figure 1(B) shows that a substantial gender wage gap of 14 log points already exists before the birth of the first child, and it increases to 21 log points seven years post-birth. Notably, the “unexplained” gap also exists before childbirth, potentially suggesting that firms might anticipate their workers’ fertility and treat men and women differently even before they have children.

Figure 1(A) and Figure 1(B) have highlighted a widening wage gap between men and women over the course of their careers. While much of the wage gap is due to gender differences in accumulated months worked (mostly after childbirth), a large proportion of the gap can also be attributed to men and women sorting into different occupations and firms. The within-job “unexplained” gap is small but statistically significant both before and after childbirth.

However, one must be cautious while interpreting these descriptive results. The “unexplained” gap might not represent the entire size of discrimination because actual experience, occupations and firms may themselves be a result of past discrimination. It is thus important to build a structural model in order to understand the drivers of differential sorting and divergent wage paths of men and women.

2.3 A glass ceiling to women’s career advancement

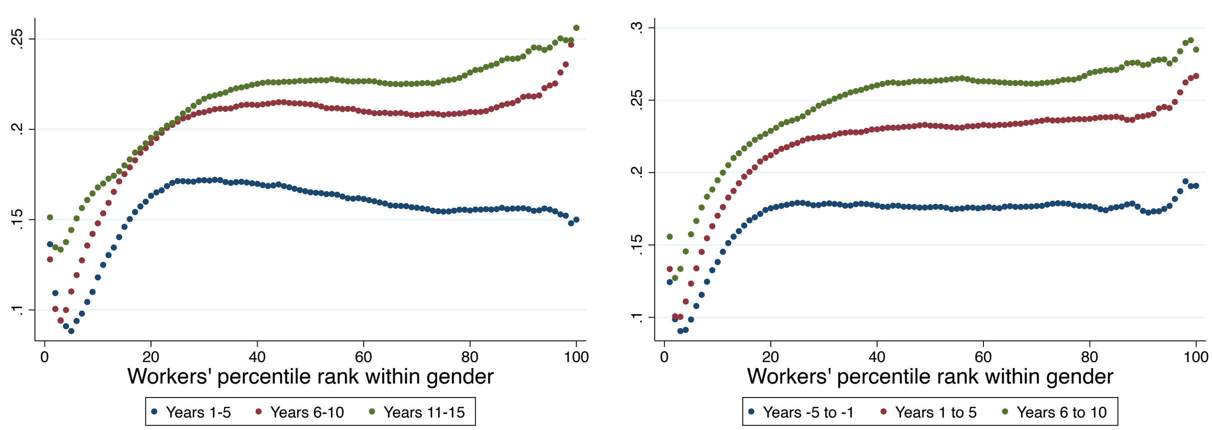
The divergent trajectories of men and women’s careers are not only evident in their mean wages, but also throughout the wage distribution. The following figures illustrate the wage gap between men and women at each percentile of their respective distributions.

Figure 2(A) shows that gender wage gaps in all percentiles shift up over years of potential experience, and a glass ceiling starts to emerge in 6 to 10 years after labor market entry. At the beginning of their careers (years 1-5), the wage gap remains similar (at

[12] Detailed field of study does not explain much of the gender wage gap after occupations are already controlled for, since occupations and fields of study are highly correlated.

FIGURE 2. Gender wage gap across the distribution

(A) Over the life-cycle (B) Around childbirth



NOTES: Each dot represents the log wage difference between men and women of the same percentile rank in their respective distribution. Panel (A) shows the gender wage gap in each percentile of the distribution by the number of years since graduation, in 5-year windows; panel (B) shows the same statistics during the 5 years before childbirth, and 5 and 10 years post-birth.

around 16 log points) for men and women above the 75th percentile; however, as their careers advance in years 6-10, the gap between the top men and top women becomes much wider (25 log points) than that at the 75th percentile (21 log points). It seems that the very best women have a difficult time climbing into the most high-paid positions. As shown in Figure 2(B), the "glass ceiling" in women's career is not only present after the birth of the first child, but is already present before childbirth (albeit to a smaller extent).[13]

In order to understand the career paths of women who are most susceptible to the glass ceiling, I look further into the sorting patterns of top men and top women over the life-cycle. I select men and women who have been at the top 25% of their respective wage distributions for at least 3 years, and plot their sorting patterns across jobs in Figure 2(A).

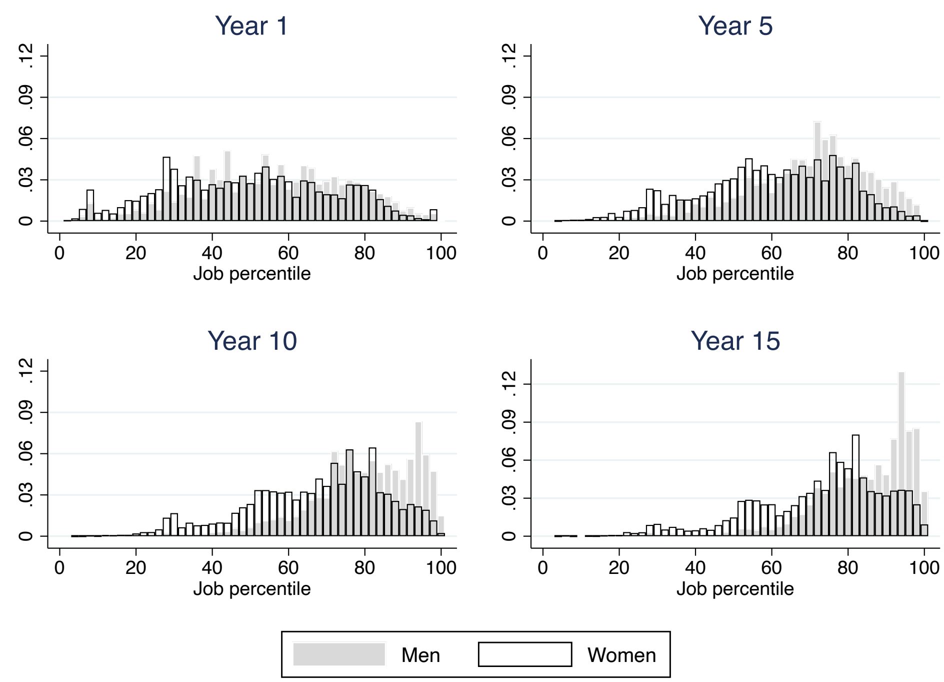
Each job is a four-digit occupation within a firm, and jobs are ranked according to their long-term average wage.[14] While men and women sort into similar jobs in the first year after labor market entry, men seem to move into higher-ranked jobs at a much faster rate than women. The distributions start to diverge at year 5, and by year 10 there is already a concentration of men in the highest-paid jobs. From years 10 to 15, women barely enter into jobs beyond the 90th percentile (which are mainly managerial positions), whereas

[13] Albrecht, Bjorklund and Vroman (2003) and Albrecht, Thoursie and Vroman (2015) document similar patterns in Sweden: there is a substantial glass ceiling for white-collar workers; it is present both before and after childbirth, and becomes more severe at older ages.

[14] The long-term average wage of each job is the average real log wage of all workers who have worked in the firm-occupation cell in all years from 1995 to 2013.

many men do.[15]

FIGURE 3. Sorting of men and women by years since labor market entry



NOTES: The histograms represent the distributions of men and women in the top quartile across all jobs in the economy. The x-axis is the job percentile, where jobs are ranked by their long-term average wage from 0 (lowest-paid) to 100 (highest-paid). Each job is a four-digit occupation within a firm.

2.4 Labor force attachment

At least part of the gaps in wages and career advancement between men and women might be attributed to gender differences in labor force attachment. The Finnish parental leave system is very generous (see Appendix C for a detailed description). Master's graduated women take on average 18 months of paid leave for each child compared to only 2 months taken by men with master's degree.

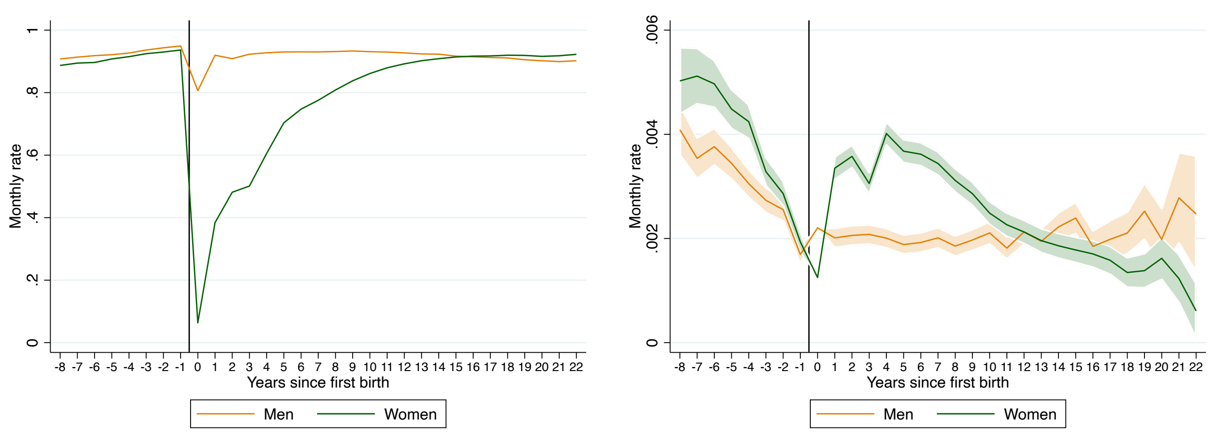
Figure 4(A) shows that men and women have similar employment rates before having children (at about 90 percent), but their labor supply diverges drastically after the birth of their first child. Virtually all women take some months off in the year of childbirth. The female employment rate increases from 6 to 38 percent one year after childbirth, but takes time to recover to its pre-birth levels since many women have a second or third child. Eventually, women's labor supply does go back to 90 percent, but only some 14

[15]The jobs beyond 90th percentile consist of business/finance/R&D managers (73%), medical doctors (9%), lawyers (3%), and business professional and engineer occupations at high-paid firms (14%).

years after the birth of the first child. In contrast, men only experience a small dip in labor supply in the year of childbirth, and do not seem to be affected afterwards.

FIGURE 4. Labor force attachment around childbirth

(A) Employment rate (PL=not working) (B) E-to-U rates



NOTES: The lines represent the regression coefficients on the number of years since first birth, with calendar year fixed effects. The outcome variables are: (A) employment indicators at a monthly level; and (B) indicators of E-to-U transitions at a monthly level, for those who are not in parental leave. Shaded areas represent 95% confidence intervals.

The large gender gap in employment rates is also driven by women's employment-to-unemployment (E-to-U) transitions after childbirth.[16] As shown in Figure 4(B), women's monthly separation rate is slightly higher than men's prior to birth, but it spikes and remain well above men's level for nine years after childbirth. This could be driven by voluntary or involuntary quits, although they cannot be distinguished in the data. Unemployment-to-employment transitions and job-to-job transitions are quite comparable between men and women, as shown in Appendix Figure A2.

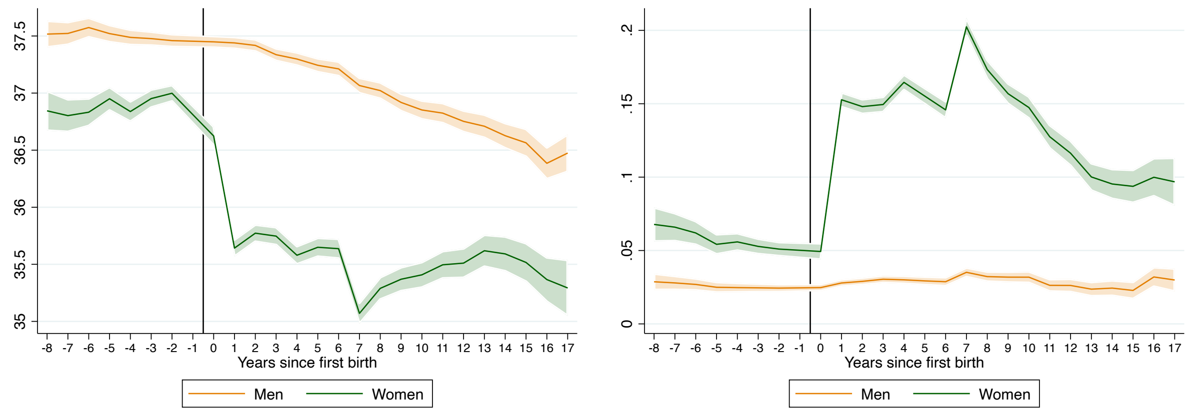
When women return to work after having children, they tend to work in different types of jobs. Figure 5(A) shows that women reduce weekly contracted hours from an average of 37 before childbirth to 35.5 immediately afterwards. Only a small proportion of educated workers have part-time jobs in Finland, but the proportion of women working part-time increases rapidly from 5 percent prior to birth to 15 percent the year after birth and remains at that level for 10 years, as shown in Figure 5(B).

Even though Finnish workers are allowed to ask for reduced hours after having children, in practice the ability to do so might depend on specific employers. Out of those

[16] Unemployment is defined as all months where the worker is not associated with any employer, whether or not he/she is actively looking for a job. Those who are on parental leave are associated with employers, so they are considered as employed for the purpose of computing E-to-U or E-to-E transition rates. See Appendix C for more details.

FIGURE 5. Demand for job amenities around birth

(A) Contracted weekly hours (B) Proportion with part-time status



NOTES: The lines represent the coefficients obtained from regressions of outcome variables on the number of years since first birth, separately for men and women, with individual fixed effects and calendar year fixed effects. Shaded areas represent 95% confidence intervals.

women who have always worked full-time before childbirth but have switched to part-time for at least one year afterwards, about 58 percent of them have to either change firms or change occupations within a firm in order to switch to part-time status. This is consistent with what Altonji and Paxson (1992) finds for the US. I use the availability of part-time work in a firm-occupation cell as part of the measure for job-specific amenities in section 2.5.

2.5 Family-friendly amenities

Workers' job choices can be driven by both wages and non-wage amenities e.g. job hazards, working conditions, stress and well-being and so on. In this paper, I focus only on the job amenities that are documented to be valued differentially by men and women, such as reduced hours, part-time work, and flexible work schedules (Goldin and Katz, 2011; Flabbi and Moro, 2012; Felfe, 2012; Goldin, 2014; Edwards, 2014; Wiswall and Zafar, 2017).[17]

I use several data sources to construct the amenity measure. The Finnish Quality of Work Life (QWL) Surveys[18] ask questions related to flexibility (positive amenities) and

[17]Commute distance is also an important amenity that women value (Le Barbanchon, Rathelot and Roulet, 2020). However, it is beyond the scope of this paper because: (i) commute distance is not observed in the data; and (ii) it requires including both worker and job locations in the value functions which significantly complicates the model.

[18]The QWL surveys are extensive studies of a representative sample of 4000 to 6000 wage or salary earners in Finland in each wave 1977, 1984, 1990, 1997, 2003, 2008 and 2013. It documents how people feel

over-working (negative amenities), listed below:

Flexibility:

- Have you agreed with the employer to work occasionally at home?
- Can you influence starting and finishing times for your work by at least 30 minutes?
- Can you use flexible working hours sufficiently for your own needs?
- Do have the possibility for brief absences from work in the middle of the working day to run personal errands?

Overwork:

- Do you sometimes work overtime without compensation?
- Have you been contacted about work outside of working hours during the last two months?
- Do you have to do more overtime work than you would like to?

The "family friendly" amenity index of each job is the first principal component of the above 7 QWL questions aggregated at 2-digit occupation level,[19] actual hours worked from the labor force survey, and the opportunity to do part-time work in a firm-occupation cell from SES. This index is used to classify job types (in the α dimension) in section 3.

Appendix Table A1 shows the factor loadings of these variables and the proportion of variation not explained by the first component. The amenity index is largely driven by the variables related to hours, as the first principal component loads more heavily on the QWL measures on overwork than on the measures of flexibility (in absolute value).[20]

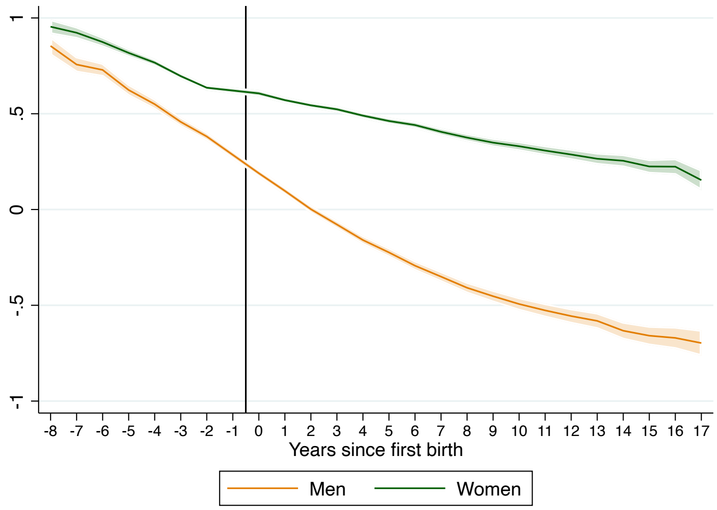
Using this amenity index, several interesting patterns emerge. First, jobs with high amenity index values are more abundant in the middle and lower end of the wage distribution (see Figure 7). Second, workers move from high- to low-amenity jobs over the life-cycle, because jobs become more demanding and require more hours and overtime as people climb the career ladder (e.g. to managerial positions). Third, Figure 6 shows that women are in jobs with slightly higher amenity index than men before childbirth, but there is a clear divergence after childbirth when women switch into high-amenity jobs and are more likely to stay there.

about their working conditions related to physical or social environment, job satisfaction, work orientation and so on.

[19] 2-digit occupations may not be detailed enough to give a fully comprehensive picture. However, the 2013 QWL is the only wave that provides information on the 3-digit level, and one cannot go into more detailed occupations due to the sample size of the surveys.

[20] The loadings of the flexibility measures are negative, possibly because highly flexible occupations (e.g. managers, science and engineering professionals etc.) also have high overtime requirements, and the index aligns more with overtime. It would be interesting to analyze how workers trade off the two dimensions of family-friendly amenities (high flexibility and low overtime hours) in future work.

FIGURE 6. Amenity index of workers' jobs around childbirth



NOTES: The lines represent the coefficients obtained from a regression of the amenity index on the number of years since first birth, separately for men and women, with worker fixed effects. Shaded areas represent 95% confidence intervals.

3 Model

Motivated by the data patterns of men and women's labor market behaviors, the model incorporates important gender differences to study both workers' and employers' decisions. I first describe the characteristics of workers and firms, and their life-cycle stages. I then explain the matching process between workers and firms and the wage determination mechanisms. Lastly, the steady-state equilibrium of the labor market is characterized.

3.1 The environment

Time is continuous and infinite. The labor market is populated by a continuum of female and male workers of measures ℓ and $(1-\ell)$, as well as a continuum of jobs of a fixed measure ι.

Workers are heterogeneous in their value for amenity ϵ, and accumulate human capital k through learning-by-doing. Upon entering the labor market, workers of gender $g \in \{m, f\}$ draw their initial skills and tastes for amenities from an exogenous discrete distribution with probability mass function $\xi_0^g(k, \epsilon)$. The model focuses only on workers' lives after graduation, and takes as given their pre-labor market decisions in human capital investment (including choices in the level of education and field of study).[21] Workers'

[21]To the extent that the field of study is highly correlated with occupation choice, the preference for different university majors is partially incorporated in the model as men and women are allowed to have

amenity preferences are assumed to be time-invariant, except that women's preference is allowed to change after childbirth.

A job is a 4-digit occupation within a firm. Jobs are heterogeneous in productivity p and amenity provision α drawn from an exogenous distribution with joint density $\varphi(p, \alpha)$. If a job is vacant, it does not produce any output and has to pay a flow vacancy cost c. Importantly, each job can match with only one worker, and employers are not allowed to search for new hires when the job is filled.[22] The distribution of jobs is fixed at $\varphi(p, \alpha)$ and there is no free entry of new vacancies.[23]

When a worker of type (k, ϵ) matches with a job of type (p, α), they produce $f(k, p)$ units of flow output (regardless of gender). Since the degree of sorting depends crucially on the supermodularity of the production function,[24] $f(.)$ is assumed to take a flexible CES form to allow for any degree of complementarity between worker and job productivities. In employment, workers receives a flow wage ω that corresponds to their bargained share of the match surplus. In non-employment, the worker's flow utility is assumed to take the form $b\,k$.[25]

The worker's human capital evolves stochastically according to a law of motion $\rho_e(k, p)$ in employment and $\rho_u(k)$ in unemployment. Human capital k is assumed to be general and transferable across different job types.[26] The skill accumulation rate in employment is allowed to depend on job productivity p, capturing the idea that workers might learn faster on the job when matched with more productive employers (Gregory, 2020), either from knowledge spillovers by more productive coworkers (Nix, 2019), or from doing more complex tasks (Caines, Hoffmann and Kambourov, 2017).

While production and human capital evolution are assumed to be symmetric between

different tastes for occupations.

[22] Assuming that the worker has an opportunity cost of accepting a job is standard in the literature, as search intensities might differ in employment and unemployment. It is thus also natural to assume that the employer incurs an opportunity cost of filling a job. While most studies assume that the marginal cost of hiring an additional worker for an already filled position is zero, my model posits that this cost is infinite. The reality might be somewhere in between where firms engage in some degree of replacement hiring (Kiyotaki and Lagos, 2007).

[23] The model can be easily extended to allow for a one-shot entry where employers can create new vacancies until the worst job makes zero profit. The important assumption is that the steady-state economy has a finite measure of jobs, and there is no vacancy creation after the initial entrance.

[24] Eeckhout and Kircher (2011) shows that the cross-partial of the production function determines the strength of sorting in realized matches, but the sign of sorting cannot be identified by wage data alone.

[25] The assumption that home productivity is proportional to the worker's skills k is common in the literature (for example in Cahuc, Postel-Vinay and Robin (2006); Flinn, Todd and Zhang (2024)).

[26] If some human capital were occupation-specific or firm-specific, that would likely amplify the role of employers in gender wage gaps. This is because firms' costly investments in job-specific human capital would yield lower returns if the worker has shorter or more interrupted employment spells.

men and women, several transition rates are allowed to be different by gender especially in fertile ages. Workers go through four age segments in life. All workers start their careers in a stage with no child (the NC stage). At an exogenous fertility rate χ, the worker has a child and enters a stage with young child (the YC stage). Every time the worker has a child, he/she will enter a parental leave (PL) stage and stay out of the labor force. Men and women might stay in the PL stage for different durations governed by exit rates η_m and η_f, upon which they can go back to their previous employers. Workers can have children repeatedly until they become non-fecund (NF) at rate γ. They retire at rate ϕ in NF stage, and new workers enter the labor market at the same rate. Within each age segment $t \in$ {NC, PL, YC, NF} of life, the search and matching process is analogous.

To simplify notation, let $\boldsymbol{x} = (k, \epsilon)$ and $\boldsymbol{y} = (p, \alpha)$ denote worker and job heterogeneity respectively.

3.2 Gender and family

The model assumes that men and women make decisions independently, even when living in the same household. This is equivalent to solving the household's problem jointly since flow utility is assumed to be linear in both employment and non-employment.[27]

Fertility is taken to be exogenous in the model mainly for tractability. While it is reasonable to expect fertility timing to depend on wages, doing so in the model might make the economy non-stationary since wages are an equilibrium object.[28]

For the same reason, gender differences in parental leave duration (governed by η^g) and separation rates (δ^g_{NC} and δ^g_{YC}) are also treated as exogenous in the model and do not respond to wages. Although men and women might make parental leave decisions together, this paper assumes that such choices are largely dictated by social norms and family leave policies, rather than being easily influenced by wage offers. As for job separations, the model does allow workers to endogenously quit in response to changes in their career prospects.[29]

[27] As noted in Dey and Flinn (2008) and Guler, Guvenen and Violante (2012), the linear utility assumption allows the household's maximization problem to be decentralized. Assuming non-linear utility will complicate the couple's joint-search problem substantially, since the spouse's values have to be taken into account when bargaining with employers.

[28] Contrary to the idea that career-oriented women might postpone child-birth, the data actually shows that women with higher initial wages (in the first three years after graduation) tend to have more children and have their first child earlier. The same positive correlation between initial wages and the number of children is observed for men too. The model abstracts from the fact that having more children is expensive.

[29] Endogenous quits alone cannot account for the substantial gender differences in quit rates observed in the data. Women's transitions from employment to unemployment might be influenced by factors beyond wages and labor market prospects, so exogenous gender differences in δ's are needed to match the transition

Men and women's tastes for family-friendly amenities are drawn from different distributions, and women's preference is also allowed to change after having children. It is important to note that these preference parameters can represent genuine taste or liking for job amenities, but they may also stem from the couple's joint optimization, reflecting the constraints in their coordination (which are not modeled).

Workers make labor supply decisions on the extensive margin (whether to accept a job or not), but they do not explicitly choose how many hours to work.[30] However, the level of family-friendly amenities α is related to the overtime demands of the job, so switching to a job with higher levels of α might entail lower hours, with implications for the hourly wage ω as described in subsection 3.4.[31]

The exogenous gender differences in this model can potentially affect the interpretation of counterfactual policies, which will be discussed in section 5. Regardless of how men and women divide child-rearing responsibilities within the household, the primary objective of this paper is to study employers' decisions as they take into account women's roles in the family.

3.3 Search and matching

At each point in time, workers can be matched to a job, unemployed, or on parental leave. The aggregate number of meetings between vacancies and searching workers is determined by a standard aggregate matching function $m(\hat{U}, V)$. This takes as inputs the total number of vacancies V and the total amount of effective job seekers $\hat{U} = U + s(1 - U)$, where U is the total number of unemployed workers and s is the search intensity in employment relative to unemployment. The matching function is assumed to be increasing in both arguments and exhibit constant returns to scale.

For ease of exposition, let $\lambda = \frac{m(\hat{U},V)}{\hat{U}\,V}$ summarize the effect of market tightness. λ is constant in a stationary equilibrium, but it is not invariant to policy, and it is important to allow it to change when evaluating interventions or counterfactual regulations.

Let $u_t^g(\boldsymbol{x})$ denote the measure of unemployed workers of gender g, age t and type $\boldsymbol{x}$, and let $v(\boldsymbol{y})$ denote the measure of vacancies of type $\boldsymbol{y}$. The joint distribution of matches between workers of type $\boldsymbol{x}$ and jobs of type $\boldsymbol{y}$ is denoted as $h_t^g(\boldsymbol{x}, \boldsymbol{y})$. While unemployed,

rates in the data.

[30] See Burdett and Mortensen (1978) and Flabbi and Mabli (2018) for search models that incorporate women's participation and hours decisions.

[31] The effect of part-time hours on human capital accumulation is partly captured by allowing skill accumulation rates to vary across different (p, α) job types.

workers randomly sample offers from the vacancies distribution, and the instantaneous rate at which an unemployed worker meets a vacancy of type $\boldsymbol{y}$ is $\lambda\, v(\boldsymbol{y})$. Similarly, employed workers meet vacancies at rate $s\lambda\, v(\boldsymbol{y})$, and vacancies meet employed workers at rate $s\lambda\, h_t^g(\boldsymbol{x}, \boldsymbol{y})$.

Upon a meeting between a worker and a job, a match will be formed if it generates positive surplus. In other words, match formation is assumed to be efficient.

Let $U_t^g(\boldsymbol{x})$ denote the lifetime value of an unemployed worker of type $\boldsymbol{x}$, $\Pi_0(\boldsymbol{y})$ denote the vacancy value of a job of type $(\boldsymbol{y})$. Let $P_t^g(\boldsymbol{x}, \boldsymbol{y})$ denote the value of joint production of a match between worker $\boldsymbol{x}$ and job $\boldsymbol{y}$. The surplus of a match is defined as $S_t^g(\boldsymbol{x}, \boldsymbol{y}) = P_t^g(\boldsymbol{x}, \boldsymbol{y}) - U_t^g(\boldsymbol{x}) - \Pi_0(\boldsymbol{y})$. A match is feasible and sustainable if the match surplus is positive.

Workers have bargaining power denoted by β and obtain a share of the match rent. Let $W_t^g(\omega, \boldsymbol{x}, \boldsymbol{y})$ and respectively $\Pi_t^g(\omega, \boldsymbol{x}, \boldsymbol{y})$ denote the value of a wage contract ω for the worker and the value for the employer, respectively. The surplus can then be written as:

$$S_t^g(\boldsymbol{x}, \boldsymbol{y}) = \underbrace{W_t^g(\omega, \boldsymbol{x}, \boldsymbol{y}) - U_t^g(\boldsymbol{x})}_{\text{Worker's share}} + \underbrace{\Pi_t^g(\omega, \boldsymbol{x}, \boldsymbol{y}) - \Pi_0(\boldsymbol{y})}_{\text{Employer's share}}.$$

The way in which wage ω splits the surplus between the worker and the employer will be discussed in the following section.

3.4 Wage determination

To define wages and renegotiations, I follow the setup in Dey and Flinn (2005) and Cahuc, Postel-Vinay and Robin (2006). Workers' wages are determined by sequential auctions. Different wages are negotiated when a worker leaves unemployment, and when counteroffers are made for an employed worker upon poaching events.

Wage bargaining with unemployed workers The starting wage $\omega_{0t}^g(\boldsymbol{x}, \boldsymbol{y})$ obtained by a type-$\boldsymbol{x}$ unemployed worker when matched with a type-$\boldsymbol{y}$ job is such that the worker receives the reservation utility $U(\boldsymbol{x})$ plus a share β of the surplus:

$$W_{0t}^g(\boldsymbol{x}, \boldsymbol{y}) = U_t^g(\boldsymbol{x}) + \beta\, S_t^g(\boldsymbol{x}, \boldsymbol{y}) \tag{1}$$

for jobs where surplus $S_t^g(\boldsymbol{x}, \boldsymbol{y})$ is positive.

Wage at job-to-job transitions When a worker of type $\boldsymbol{x}$ encounters an alternative job package $\boldsymbol{y}'$ that produces more surplus than her current job, she will switch jobs with a wage $\omega_{1t}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')$ such that the value she receives at the new job $\boldsymbol{y}'$ is $W^g_{1t}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')$. In this scenario, the worker extracts the maximum value from the incumbent match $P^g_t(\boldsymbol{x},\boldsymbol{y}) - \Pi_0(\boldsymbol{y})$ plus a β share of the surplus difference:

$$W^g_{1t}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') = P^g_t(\boldsymbol{x},\boldsymbol{y}) - \Pi_0(\boldsymbol{y}) + \beta\left[S^g_t(\boldsymbol{x},\boldsymbol{y}') - S^g_t(\boldsymbol{x},\boldsymbol{y})\right] \tag{2}$$

Wage renegotiation upon poaching If the poaching job $\boldsymbol{y}'$ generates a match surplus below that of the incumbent job, i.e. when $S^g_t(\boldsymbol{x},\boldsymbol{y}') < S^g_t(\boldsymbol{x},\boldsymbol{y})$, the worker will stay in the same job. Incumbent employers will respond to outside offers and update wages only when there is a credible threat – when either the worker or the employer will credibly separate if they do not obtain an improved offer. In other words, wages will be re-negotiated when the poaching firm offers a value greater than what the worker currently receives. In this case, wages will be updated to $\omega_{2t}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})$ such that the worker receives an updated value $W^g_{2t}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})$ at the incumbent job $\boldsymbol{y}$:

$$W^g_{2t}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) = P^g_t(\boldsymbol{x},\boldsymbol{y}') - \Pi_0(\boldsymbol{y}') + \beta\left[S^g_t(\boldsymbol{x},\boldsymbol{y}) - S^g_t(\boldsymbol{x},\boldsymbol{y}')\right] \tag{3}$$

Note that when a worker's human capital changes from k to k', her wage does not update until there is a credible outside option. Please refer to Appendix D for details of the workers' values.

3.5 Value functions

In order to define an equilibrium, I will describe the value functions and the distributions of workers and jobs across employment states and life stages. These define the decision rules for each agent.

3.5.1 Value in unemployment

In the "no child" stage of life, the utility of an unemployed worker is:

$$\begin{aligned}\Big(r + \sum_{k'}\rho_u(k'|k) + \chi + \gamma\Big)\, U^g_{NC}(\boldsymbol{x}) = b\,k + \sum_{k'}\rho_u(k'|k)\, U^g_{NC}(k',\epsilon) + \chi\, U^g_{PL}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) \\ + \sum_{\boldsymbol{y}}\lambda\, v(\boldsymbol{y})\;\beta\,[S^g_{NC}(\boldsymbol{x},\boldsymbol{y})]^+ \end{aligned} \tag{4}$$

r is the risk-free interest rate, and $[S]^+$ denotes the maximum operator $\max\{S, 0\}$. The transition matrix for the worker's human capital level in unemployment is given by the law of motion $\rho_u(k'|k)$.

The worker is subject to life-cycle shocks. When an unemployed worker has a child at rate χ, he/she exits the labor market and enters a period of parental leave and do not conduct job search in the PL stage. When parental leave terminates at rate η^g, the unemployed worker enters the labor market and resumes job search in unemployment in the "young child" (YC) stage. At any point in life, the worker ages at rate γ, upon which he/she enters a non-fecund period. The details of the unemployment values in PL, YC and NF stages are described in Appendix E.

3.5.2 Value of vacancy

A vacant job could potentially hire a male or female worker of any age $t \in \{NC, YC, NF\}$. The value of a vacancy of type $\boldsymbol{y}$ is:

$$\Big(r + \lambda\, U + s\lambda\,(1-U)\Big)\,\Pi_0(\boldsymbol{y}) = -c + \sum_{t,g,\boldsymbol{x}} \lambda\, u_t^g(\boldsymbol{x})\,(1-\beta)\Big[S_t^g(\boldsymbol{x},\boldsymbol{y})\Big]^+ \\ + \sum_{t,g,\boldsymbol{x},\boldsymbol{y}'} s\lambda\, h_t^g(\boldsymbol{x},\boldsymbol{y}')\,(1-\beta)\Big[S_t^g(\boldsymbol{x},\boldsymbol{y}) - S_t^g(\boldsymbol{x},\boldsymbol{y}')\Big]^+ \tag{5}$$

c is the flow cost of keeping a vacancy open, and U denotes the aggregate unemployment. Job vacancies have the opportunities to meet unemployed and employed workers of any age, gender, productivity and preference types. Since employers have capacity constraints, the option value of waiting Π_0 is typically positive.

3.5.3 Joint value of a match

In the "no child" stage, the joint value of a match between worker $\boldsymbol{x}$ and job $\boldsymbol{y}$ is:

$$\Big(r + H(k,p) + \delta_{NC}^g + \chi + \gamma\Big)\,P_{NC}^g(\boldsymbol{x},\boldsymbol{y}) = \underbrace{(1-\tau)\,f(k,p)}_{\text{after-tax flow output}} + \underbrace{q(\epsilon,\alpha)}_{\text{value for amenities}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\,\widetilde{P}_{NC}^g(\boldsymbol{x}',\boldsymbol{y})}_{\text{HC accumulation}} \\ + \underbrace{\delta_{NC}^g\Big(\Pi_0(\boldsymbol{y}) + U_{NC}^g(\boldsymbol{x})\Big)}_{\text{exogenous separation}} + \underbrace{\chi\,\widetilde{P}_{PL}^g(\boldsymbol{x},\boldsymbol{y})}_{\text{fertility}} + \underbrace{\gamma\,\widetilde{P}_{NF}^g(\boldsymbol{x},\boldsymbol{y})}_{\text{ageing}} \\ + \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\;\beta\Big[\underbrace{S_{NC}^g(\boldsymbol{x},\boldsymbol{y}')}_{\text{poaching job surplus}} - \underbrace{S_{NC}^g(\boldsymbol{x},\boldsymbol{y})}_{\text{current job surplus}}\Big]^+ \tag{6}$$

The match between worker of human capital k and job of productivity p produces $f(k,p)$ units of flow output, regardless of gender. There is a proportional tax τ on the flow output to finance parental leave benefits. The worker enjoys a flow utility that is a function of his/her value for amenities ϵ and the level of amenity provision at the job α.

The transition matrix for the worker's human capital in employment is given by the law of motion $\rho_e(k'|k,p)$, where the total rate of HC change is denoted by $H(k,p) = \sum_{k'} \rho_e(k'|k,p)$. Upon exogenous separation δ^g_{NC}, the match dissolves and the worker and the employer both receive their outside options. The worker searches on-the-job, and employers Bertrand-compete for the worker.

All matches are efficient, and existing match are allowed to endogenously dissolve if the joint value of the match falls below the sum of the agents' outside options in separation. There could be endogenous quits when human capital level x changes and at any age segment t in life:

$$\widetilde{P}^g_t(\boldsymbol{x},\boldsymbol{y}) = \max\left\{P^g_t(\boldsymbol{x},\boldsymbol{y}),\ \Pi_0(\boldsymbol{y}) + U^g_t(\boldsymbol{x})\right\}, \qquad t = \{NC, PL, YC, NF\}$$

3.5.4 Parental leave

When a worker has a child, several changes take place. The woman's utility from amenities changes from $q(\epsilon,\alpha)$ to $q^f_{YC}(\epsilon,\alpha)$, whereas the men's value stays the same. Exogenous separation rates also change from δ^g_{NC} to δ^g_{YC}. There is no human capital accumulation and no job search in the PL stage. The joint value in parental leave is:

$$\begin{aligned}(r + \delta^g_{YC} + \eta^g + \gamma)\, P^g_{PL}(\boldsymbol{x},\boldsymbol{y}) = (1-\tau) \quad & \underbrace{R\, f(k,p)}_{\text{reduced flow output}} \quad + \quad \underbrace{q^g_{YC}(\epsilon,\alpha)}_{\text{value for amenities}} \quad + \underbrace{\delta^g_{YC}\Big(\Pi_0(\boldsymbol{y}) + U^g_{PL}(\boldsymbol{x})}_{\text{exogenous separations}}\Big) \\ & + \underbrace{\eta^g\, \widetilde{P}^g_{YC}(\boldsymbol{x},\boldsymbol{y})}_{\text{PL ends}} + \underbrace{\gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y})}_{\text{ageing}} \end{aligned} \tag{7}$$

Mimicking the institutional settings in Finland as closely as possible, the model assumes the following. First, the worker goes into parental leave immediately after having a child, and gets paid a wage that is fully funded by the government for the whole duration of leave. Second, the worker on leave enjoys job protection and the employer has to keep the job available for when he/she returns. Third, the job still produces a flow output when the worker is absent, but production is slashed to a ratio R proportion of its previous amount.

One could think of parameter R as a reduced-form way of capturing various challenges and adjustment costs faced by firms whenever a worker goes on parental leave. Even though Finnish employers do not face direct costs of financing employees' wages while on leave, they may still encounter difficulties and costs in finding a replacement worker and/or coordinating schedules of existing workers to keep production going, potentially at a lower productivity. Ginja, Karimi and Xiao (2023) quantifies these adjustment costs experienced by firms in Sweden.[32]

From a modeling perspective, the employer continues production in this period but does not hire new workers; conceptually one could think of the job as being covered by co-workers working more hours. Abstracting from hiring temporary workers would simplify the model in parental leave stage as one does not need to keep track of any new workers being hired (and then laid off) in steady state balance flows.[33]

Workers on leave are by default associated with their previous employers (in both the data and model), but can separate from their employers exogenously or endogenously in the parental leave period. Women and men exit parental leave at rate η^f and η^m respectively, upon which unemployed workers start searching for jobs and employed workers go back to pre-birth employers. The worker can have another child any time during fertile ages (including during parental leave). Upon having another child while employed, the worker will go into parental leave again.

The government runs a balanced budget. The tax rate τ is set such that total government transfers to workers on parental leave are equal to the total tax revenues collected from all job matches in stationary equilibrium:

$$\sum_{g,\boldsymbol{x},\boldsymbol{y}} \omega^g_{0,PL}(\boldsymbol{x},\boldsymbol{y})\, h^g_{PL}(\boldsymbol{x},\boldsymbol{y}) = \sum_{g,\boldsymbol{x},\boldsymbol{y},t} \tau\, f(k,p)\, h^g_t(\boldsymbol{x},\boldsymbol{y})$$

where $\omega^g_{0,PL}(\boldsymbol{x},\boldsymbol{y})$ denotes the flow wage in PL stage.

The transition parameters and preference parameters in "young child" (YC) stage are the same as in PL stage, and one should think of these two stages as the period where workers have young children at home. The only difference is that individuals in PL stage are matched with some employers but are not working, whereas those in YC stage are actively participating in the labor force.

In stage NF, individuals are non-fecund and will not have any additional child. In this period, men and women have the same separation rate δ, and retire at rate ϕ.

[32]Ginja, Karimi and Xiao (2023) finds that firms hired temporary workers and increased incumbents' hours when parental leave was extended by 3 months in Sweden. Even though firms did not have to pay wages to the person on leave, the total wage bill cost of the re-organization was on average equivalent to 10 full-time months for each additional worker on extended leave.

[33]This means that the employer would split the match surplus with the co-workers, not with workers on leave. I do not solve for equilibrium wages of the over-working co-workers. Workers on leave receive benefits outside the system of worker-job pairs – they get benefits directly from the government.

The details of the match values in YC and NF stages are described in Appendix E.

3.6 Steady-state balance flow conditions

In equilibrium all agents follow their optimal strategy. Denote the measure of workers of gender g in age segment $t \in \{NC, PL, YC, NF\}$ as m_t^g. The total measure of women of all ages should add up to $\ell^f = \ell$, and men to $\ell^m = 1 - \ell$.

$$m_{NC}^g + m_{YC}^g + m_{PL}^g + m_{NF}^g = \ell^g \tag{8}$$

Also, the flows into and out of each age segment should balance.

$$\chi \, (m_{NC}^g + m_{YC}^g) = (\gamma + \eta^g) \, m_{PL}^g \tag{9}$$

$$\eta^g \, m_{PL}^g = (\chi + \gamma) \, m_{YC}^g \tag{10}$$

$$\gamma \, (m_{NC}^g + m_{YC}^g + m_{PL}^g) = \phi \, m_{NF}^g \tag{11}$$

The equilibrium distribution of vacancies and matches will satisfy the following accounting equation:

$$v(\boldsymbol{y}) + \sum_{g,\boldsymbol{x},t} h_t^g(\boldsymbol{x}, \boldsymbol{y}) = \varphi(\boldsymbol{y}), \quad t \in \{NC, YC, PL, NF\} \tag{12}$$

where the total measure of all jobs is fixed at $\sum_{\boldsymbol{y}} \varphi(\boldsymbol{y}) = \iota$.

The equilibrium distribution of workers is such that flows into and out of any worker stock must balance for each worker type $(g, \boldsymbol{x}, t)$, in employed or unemployed state, across all job types (if employed). Appendix F describes the steady-state balanced flow conditions.

3.7 Definition of equilibrium

A stationary equilibrium is a tuple of value functions $\{U^m, U^f, P^m, P^f, \Pi_0\}$ together with a distribution of male and female workers across employment states and across job types $\{u^m, u^f, h^m, h^f\}$ as well as a distribution of job vacancies v such that:

(i) The value functions satisfy Bellman Equations (4) to (7) and those in Appendix E.

(ii) The distributions $\{u^m, u^f, h^m, h^f, v\}$ are stationary given the transitions implied by the value functions, and satisfy balanced flow conditions (8) to (12) and flow equations in Appendix F.

(iii) Equilibrium wages are determined by surplus sharing rules defined in (1) to (3).

Note that the equilibrium values and allocations (points (i) and (ii) above) can be solved without making any reference to wages, just like in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). This is because utility is transferable between the worker and employer, so joint values and surpluses do not depend on wages. Moreover, match formation and worker mobility decisions are determined only by the surpluses or difference in surpluses between two jobs, so the equilibrium worker and job allocations also do not depend on wages. The advantage of this transferable utility framework is that it makes the model very tractable, and the computation of the equilibrium fairly feasible.

There is no analytical solution to the equilibrium. Given the richness in the model structure designed to reflect characteristics of the Finnish labor market, proving the existence and uniqueness of the equilibrium is virtually impossible. I solve for the equilibrium numerically. For a given set of model parameters and smoothing shocks (see subsection 3.8), I show that the solution always converges to the same equilibrium values and distributions when starting with different initial guesses of the equilibrium objects.

3.8 Smoothing shocks

Since worker types and job types are discrete while surplus values are continuous, there is no guarantee that the solution algorithm always converges. This is due to the possibility that two jobs may offer very similar surplus values for a given worker type, for example when one job provides lower productivity with greater amenities and another offers higher productivity with less amenities. An infinitesimal change in the match values might alter the ranking of these jobs, resulting in a discrete shift in workers' mobility and the steady-state flows across jobs. Similarly, when a match has a surplus value close to zero, a slight change in the value function could cause the match to shift from being unacceptable to acceptable, or vice versa.

In order to break the ties between any two jobs and smooth the match formation decisions, I introduce an i.i.d shock z upon each job arrival. An unemployed worker would accept a job with surplus S if $S + z > 0$. Similarly, an employed worker with match surplus S will move to a new job with match surplus S' if $S' + z > S$. The shock z is assumed to be Logistically distributed with mean zero.

This setup is similar to Borovickova and Shimer (2024), although the additiveness assumption and logistic distribution of the shock in my model makes the solution much simpler. Please refer to Appendix G for details of the value functions and steady-state balanced flows with the smoothing shocks.

In practice, introducing the smoothing shock z is a technical step that ensures the existence of an equilibrium. The variance of the shock is fixed to a small value so that it does not affect the quantitative results of the model.

4 Estimation

In this section, I estimate the model using Simulated Method of Moments (SMM).[34] To this aim, I obtain a vector of moments from N individuals in the data, $\hat{m}^D = \frac{1}{N}\sum_{i=1}^{N} m_i$, for example mean wages out of unemployment in the first five years after graduation, etc. Model counterparts to these moments, $\hat{m}^S(\theta) = \frac{1}{M}\sum_{j=1}^{M} m_j^D$, are obtained from M simulated lives from the model based on a parameter vector θ. The estimation involves finding the vector θ that brings the simulated moments as close as possible to the data moments, i.e. minimizing the criterion function

$$L(\theta) = (\hat{m}^D - \hat{m}^S(\theta))^T \ \hat{W}^{-1} \ (\hat{m}^D - \hat{m}^S(\theta))$$

where $\hat{W}$ is a weighting matrix.

Key parameters of interest are outlined below.

4.1 Model specification

Individual records are aggregated to the monthly level in the data, so I set the length of a model period to be one month, assuming that at most one event can occur within each period.[35]

Human capital of the worker can take one of K discrete values $k \in \{k_1, k_2, ..., k_K\}$ and $0 < k_1 < k_2 < ... < k_K$. Human capital accumulation is assumed to take the form

$$\rho_e(k_{i+1} \,|\, k_i, p) = d_1 + d_2\, p, \qquad i \in \{1, 2, ..., K-1\}.$$

That is, every period an employed worker moves up by one category of human capital at a rate that is linear in his/her job productivity p. This captures the idea that workers might learn faster on the job when matched with more productive employers. The worker's human capital is assumed to stay unchanged while he/she is not working.[36]

[34]See for example McFadden (1989) and Pakes and Pollard (1989). Constructing the likelihood function for this model is intractable.

[35]This is an approximation of the continuous-time model. The approximation is reasonable given that all the transition probabilities are small, including E-to-U, U-to-E, job-to-job, and transitions into and out of parental leave. The estimates for human capital growth are also very small.

[36]The estimation can easily allow human capital to depreciate when non-employed, though there is no reason to think it would substantially change the counterfactuals.

When non-employed, the worker receives a flow utility of home production that takes a proportional form $b\,k$. While employed, the flow output produced by the match is specified by a flexible CES production function:

$$f(k,p) = A\left[a\,k^{\sigma} + (1-a)\,p^{\sigma}\right]^{\frac{1}{\sigma}}.$$

Central to the model is the matching between workers and jobs, which is intimately related to the production function. A CES function allows for various degrees of complementarity between the worker's human capital and the employer's productivity, governed by the estimated value of σ. If $\sigma < 1$, the production function is supermodular, whereas if $\sigma > 1$, it is submodular. The CES function is a generalization of the commonly used multiplicative form $k\,p$ (for example in Cahuc, Postel-Vinay and Robin (2006) and Flinn, Todd and Zhang (2024)).

Men and women draw their values for amenities ϵ^m and ϵ^f from normal distributions $N(\mu_m, 1)$ and $N(\mu_f, 1)$ respectively.[37] In the "no child" stage, the flow utility from amenities takes the simple form $q(\epsilon^g, \alpha) = \epsilon^g\,\alpha$. Women's value increases by M in motherhood, so that $q^f_{YC} = (\epsilon^f + M)\,\alpha$ in YC and PL stages, whereas men's values stay the same $q^m_{YC} = \epsilon^m\,\alpha$.

Finally, I assume the matching function has an elasticity of 0.5 and takes the functional form (see Petrongolo and Pissarides (2001)):

$$m(\hat{U}, V) = \vartheta\sqrt{\hat{U}\,V}$$

where effective job seekers $\hat{U} = U_{NC} + s_U(U_{YC} + U_{NF}) + s_E(1 - U_{NC} - U_{YC} - U_{NF})$. I allow search in unemployment to be different in early and late stages in life. The search intensity for the unemployed in NC stage is normalized to one, and that of the unemployed in YC and NF stages will be s_U. The relative search intensity of the employed is s_E and does not vary over the life-cycle.

The initial human capital of male and female workers follow exponential distributions $\xi^g_0(k) \sim \text{Exponential}(\xi_g)$. The initial human capital distribution is assumed to be the same within each worker preference type.

[37] The standard deviations of the distributions are normalized to 1 because of identification issues discussed in Taber and Vejlin (2020): in a standard search model with worker bargaining and preference for job amenities, the bargaining power parameter cannot be separately identified from the scale of utilities.

4.2 Job types

Job productivity types are discrete and not directly observed from the data. Since sorting and wages are endogenous to worker and job types in this model, the ideal estimation method would involve updating job classifications in iterative steps using an Expectation-Maximization (EM) algorithm such as that in Lentz, Piyapromdee and Robin (2023).[38] However, the model in this paper does not yield analytical moments or closed-form expressions for the likelihood function. The estimation process must rely on simulation-based methods, which makes the iterative procedure computationally infeasible.

Therefore, I follow Bonhomme, Lamadon and Manresa (2019) (henceforth BLM) and pre-classify job types in one step through k-means clustering. BLM shows that firm classes are identified from within-firm wage CDFs when there are sufficiently many workers per firm. However, that is not the case in my setting because many jobs (firm-occupation cells) have less than 5 workers.[39] Instead of wage distributions, I use the long-term average wage (between 1995 and 2013) in each firm-occupation cell to classify job types.

Even though job classes cannot be identified from average wages alone under NAM (negative assortative matching), they are identified under PAM (positive assortative matching) because long-run average wages are monotonic in job productivity in this case.[40] Assuming PAM ex-ante for the estimation is less ideal. However, researchers have found overwhelming evidence of positive sorting between workers and firms in many countries.[41] Moreover, an advantage of using average wages to classify jobs is that there is a clear hierarchy of job types, which is useful when analyzing career progression over the life-cycle.

I estimate seven productivity clusters, where the support of the distribution is normalized so that the bottom type takes a productivity value of 1. Workers' human capital levels have the same support. Table A2 provides summary statistics about the job productivity

[38] In the E step, one would calculate posterior probabilities for a given set of model parameters and job type classifications. In the M step, model parameters are updated by maximizing the expected log likelihood of observing the data. The job classification is then updated given the parameter estimates, and the process repeats.

[39] The research question calls for a granular definition of jobs, since career progression of men and women might involve both moving up the occupational ranks within a firm as well as advancing toward better firms.

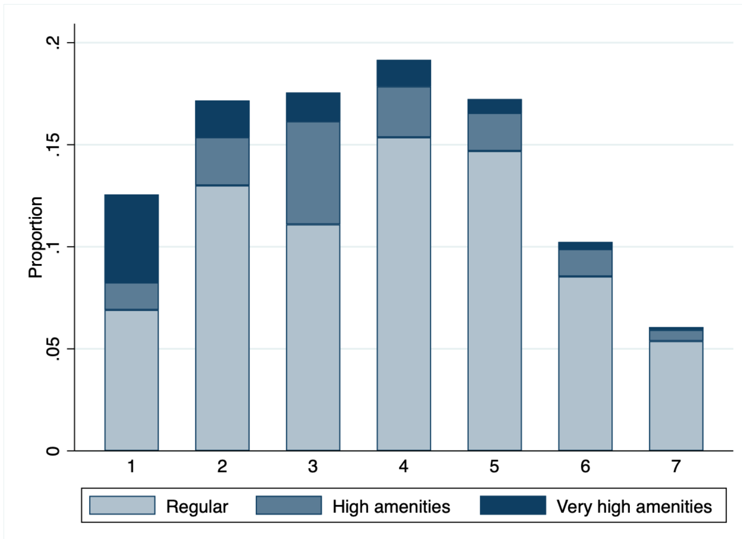
[40] Intuitively, highly-productive workers sort into highly-productive jobs under PAM, so average wages are increasing in job productivity type in steady-state equilibrium. This is shown in BLM's Supplemental Materials for a variation of the model of Shimer and Smith (2000) which is very similar to mine.

[41] For example in the US (Lopes de Melo, 2018; Bonhomme, Holzheu, Lamadon, Manresa, Mogstad and Setzler, 2023), Sweden (Bonhomme, Lamadon and Manresa, 2019), Denmark (Bagger and Lentz, 2018), Germany (Hagedorn, Law and Manovskii, 2017), Italy (Bartolucci, Devicienti and Monzón, 2018) amongst others.

categories.

Each job is also assigned an amenity type based on its amenity index (constructed in subsection 2.5). There are three categories of amenity provision: very high (more than 1 s.d. above average), high (between 0.5 and 1 s.d. above average) and regular jobs (the rest). The empirical distribution of jobs across both productivity and amenity dimensions are shown in Figure 7.

FIGURE 7. Distribution of jobs by productivity and amenities



4.3 Estimation method and identification

Given the above specification, I estimate two sets of parameters in an iterative procedure. The first set of parameters $\mathbf{\Lambda} = (\boldsymbol{\delta}_t^g, \vartheta, s_U, s_E)$ includes a vector of exogenous separation rates $\boldsymbol{\delta}_t^g$ and parameters from the matching function. The second group includes "core" model parameters characterizing human capital processes, production functions, bargaining, preferences, and initial distributions, denoted by $\mathbf{\Theta} = (d_1, d_2, A, a, \sigma, \beta, b, \mu_m, \mu_f, M, \xi_m, \xi_f)$.

Note that separation rates, job-finding rates and job-to-job transition probabilities in the model depend on equilibrium surplus values and the equilibrium distribution of vacancies, and consequently cannot be obtained independently outside of the model. However, given the equilibrium surpluses, parameters in $\mathbf{\Lambda}$ are directly related to workers' transitions in and out of work and between jobs. Therefore, $\mathbf{\Lambda}$ can be identified given $\mathbf{\Theta}$. Following Meghir, Narita and Robin (2015), I estimate the two groups of parameters using an iterative procedure that significantly reduces estimation time. For details of the estimation procedure and computation of standard errors, please refer to Appendix H.

Several points are worth noting regarding identification. The main assumptions and arguments are as follows.

Discrimination Based on fertility-related turnover concerns, employers' gender-specific wage-setting and job allocation decisions depend on both $\mathbf{\Lambda}$ and $\mathbf{\Theta}$ parameters in the model. These decisions can be distinguished from taste-based discrimination with the following intuition. After people become non-fecund (at age 45 on average), men and women face very similar model primitives as they no longer have children. Therefore, any gender wage gap after age 45, after conditioning on human capital histories up to that point, cannot be attributed to statistical discrimination based on child-related concerns. Any taste-based discrimination will be captured in the model residual, and cannot be separated from initial productivity differences between men and women (ξ_0^m and ξ_0^f).

Job types Once job clusters are estimated, they are taken as observed. This significantly simplifies the identification problem, as mobility patterns across high- and low-amenity jobs (within the same productivity category) help to identify workers' preferences for amenities, and mobility patterns across high- vs. low-productivity jobs (within the same amenity category) reveal the extent of production complementarity.

Preferences Amenity preference parameters μ_m and μ_f govern workers' mobility patterns across jobs of high- and low- amenity types, and do not affect the production of output. Therefore, the proportion of female workers in high-amenity jobs helps pin down the magnitude of μ_f relative to μ_m. The increase in value for amenities during motherhood M is closely linked to the proportion of women who switch into high-amenity jobs immediately after childbirth.

Amenity preferences also affect wages through compensating differentials. The wage differences between high- and low- amenity jobs of the same productivity type can help to identify μ_f and μ_m. However, it is important to note that worker's human capital is not observed in the data, so the wage differences include two opposing effects: first, compensating differentials push wages down in high-amenity jobs; and second, high-amenity jobs are more able to attract high-HC workers, making the average wage higher since the workers there are more positively selected. Both effects are in the model, and corresponding data moments would speak to which force dominates.

Human capital Human capital growth parameters d_1 and d_2 do not have a direct data counterpart since the assignment of workers to jobs is not random. However, with the aid of the full equilibrium structure of the model, these parameters can be related to the following aspects of the data. When a worker goes through an unemployment spell in the model, she falls off the job ladder and loses any "search capital" accumulated through job-to-job transitions. However, human capital is general and she will carry her accumulated experience to the next job. Comparing the wages immediately following a transition from unemployment to employment (UE wages) at different points of the life-cycle can inform us of the average human capital growth rate d_1 in the economy.

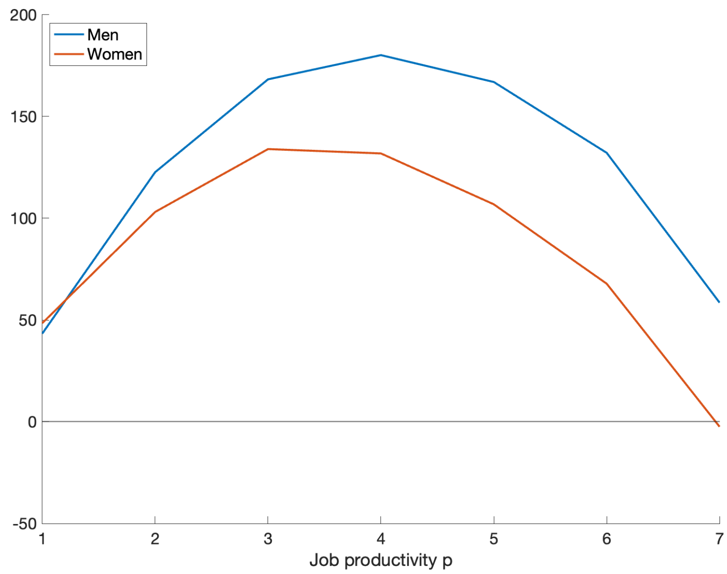
Moreover, human capital growth in each productivity category p is related to within-job wage growth in jobs of high- versus low-productivity types. Although wage gains within a job also depend on renegotiations triggered by poaching firms, the amount of contact with poachers is disciplined by s_E and ϑ that are pinned down in the previous step. Therefore, the remaining within-job wage growth could be attributed to human capital growth.

Production function Key to identification of production function parameters is the sorting of men and women across jobs. When production is very complementary (σ very small or negative), the marginal return of employing a high-type worker is considerably higher for high-productivity jobs. In the presence of a capacity constraint of a firm, this implies that the match surplus might not be monotonically increasing in job productivity (Eeckhout and Kircher, 2011), since high-productivity jobs have a much higher option value of waiting for a better match.

Indeed, the values of match surplus might be an inverted-U shape (as shown in Figure 8), or even decreasing in job productivity for a low-type worker. The example in Figure 8 shows that with production complementarity, the medium-skilled workers are best matched with middle-level jobs where the surpluses peak. Top jobs (category 7, mainly managers) generate relatively low surpluses with mediocre workers, and this is more severe for women as they have higher turnovers and generate less surplus in general. High vacancy values of the top jobs imply that these employers might turn off matches with women even though they might still match with equally skilled men.

Consider the contrary case where production is perfectly substitutable ($\sigma = 1$), then there are no productivity gains from sorting compared to random matching. Surpluses will be monotonically increasing in job productivity for any given worker type. Since match values are typically lower for women than men, it would imply that low-productivity jobs are the ones that stop matching with women, and we would see different sorting patterns of men and women vis à vis the case where production is complementary.

FIGURE 8. An example of surplus values of medium-skilled workers in NC stage



NOTES: The solid lines plot the surplus values of a male and female worker in "no child" stage across jobs of different productivity levels. The man and woman have the same amenity preference and same productivity (both of skill type k_3). The production function in this example assumes complementarity between worker and job productivities, with $\sigma = -0.9$.

Labor share Relative productivity of labor (parameter a) is closely related to human capital parameters and wage growth over the life-cycle. When human capital appreciates, production grows more when a is high. Although both d_1, d_2 and a are positively related to wage growth moments, they could have opposite implications for UE wage levels. The intuition is that when a increases, all jobs are much better off matching with high-HC workers when production is complementary, and top jobs are actually worse off matching with low-type workers given the increased option value of hiring high-types. In contrast, an increase in d_1 or d_2 invariably raises surpluses and UE wages of all matches. As a result, in early career stages when most workers do not have much human capital, we will see lower UE wages when a increases but higher UE wages when d_1, d_2 increase. The extent of these effects depends on the strength of complementarity.

Calibrated parameters I calibrate the life-cycle Poisson parameters. Fertility rate χ is calibrated to match the average number of children workers have (both men and women have 1.7 children on average). Ageing rate γ is set to match the number of years between graduation and age 45, and retirement rate ϕ is set so that individuals retire at age 60 on average. The rates at which parental leave ends for men and women, η_m and η_f, are calibrated to match the average length of parental leave taken for each child by men and women respectively.

TABLE 1. Parameter Estimates

Parameters		Estimates	S.E.	Top 3 Sensitivity Moments
(A) Θ parameters				
Complementarity	σ	-14.491	1.167	M9, M4, M1
Relative productivity	a	0.848	0.021	M2, M6, M5
TFP	A	29.526	1.392	M1, M4, M3
Baseline HC rate	d_1	0.001	0.002	M2, M6, M5
Proportional HC rate	d_2	0.010	0.003	M5, M6, M2
Men's value for amenities	μ_m	0.767	0.025	M10, M7, M12
Women's value for amenities	μ_f	0.883	1.060	M10, M7, M12
Preference increase in motherhood	M	1.815	1.923	M12, M10, M11
Worker's bargaining	β	0.519	0.016	M2, M4, M5
Home productivity	b	5.109	0.897	M1, M11, M2
Initial distribution - men	ξ_m	2.201	0.936	M8, M3, M1
Initial distribution - women	ξ_f	2.798	1.013	M8, M3, M1
(B) Λ parameters				
Women's separation rate in NC	δ_{NC}	0.011	0.001	M15, M11, M12
Women's separation rate in YC	δ_{YC}	0.015	0.012	M15, M11, M9
Men's separation rate	δ	0.008	0.002	M15, M11, M4
Matching efficiency	ϑ	0.110	0.007	M14, M16, M11
Relative search intensity in unemployment	s_U	0.687	0.423	M14, M11, M9
Relative search intensity in employment	s_E	0.526	0.214	M16, M9, M10

NOTES: Column "Top 3 Sensitivity Moments" reports the three most important moments for each parameter in estimation based on the sensitivity measure. M1, ..., M16 denote the 16 groups of targeted moments:

M1: Mean wages by gender (every 3 years)
M2: UE wages by gender (every 5 years)
M3: Initial gender wage gap (in year 1)
M4: Standard deviation (SD) of wages by gender (every 5 years)
M5: Within-p wage growth by gender from years 1-5 to years 21-25
M6: Within-p UE wage growth by gender from years 1-5 to years 21-25
M7: Wage difference between high vs. low amenity jobs (every 5 years)
M8: Initial distribution by gender (in year 1)
M9: Distribution of men and women across job productivity p (every 5 years)
M10: Proportion in high-α jobs by gender (every 5 years)
M11: Employment rate around childbirth by gender (every 5 years)
M12: Gender difference in high-α proportions around birth (every 5 years)
M13: Gender wage gap around birth (every 5 years)
M14: UE transition rates by gender in each life-cycle stage
M15: EU transition rates by gender around birth (every 5 years)
M16: Mean EE transition rates by gender

Other calibrated parameters include R and c. The reduction in flow production R during parental leave is calibrated to the adjustment costs of extended parental leave estimated in Ginja, Karimi and Xiao (2023). The vacancy cost c is calibrated to that in Lise, Meghir and Robin (2016). The monthly risk-free interest rate r is set to 0.012. The list of calibrated parameters are shown in Table A3.

4.4 Results

The complete set of parameter estimates is presented in Table 1. The last column of Table 1 reports the sensitivity analysis (Andrews, Gentzkow and Shapiro, 2017) and shows the three most important moments for each parameter in estimation. The sensitivity analysis is in line with the identification arguments.

The human capital estimates show that workers' skill accumulation rate is positively related to job productivity – workers' human capital upgrades much faster when they work at highly productive firms. The estimates imply that in the bottom job productivity category, human capital appreciates at the rate of 0.011, whereas at the top of the distribution the rate is 0.034 (since p ranges from 1 to 3.28). There will be a divergence in human capital levels of men and women over time, not only because men spend more time working and accumulating skills, but also because men are more represented at top jobs that offer better learning opportunities.

The estimate of σ implies an elasticity of substitution of 0.065, reflecting a high degree of complementary between worker and firm productivity.[42] Given the substantial gains from sorting, there are strong incentives for high-productivity workers to seek out high-productivity jobs, while low-HC workers gravitate toward lower-end jobs. Moreover, assortative matching becomes more pronounced over the life-cycle as mismatched workers gradually find matched jobs through on-the-job search.

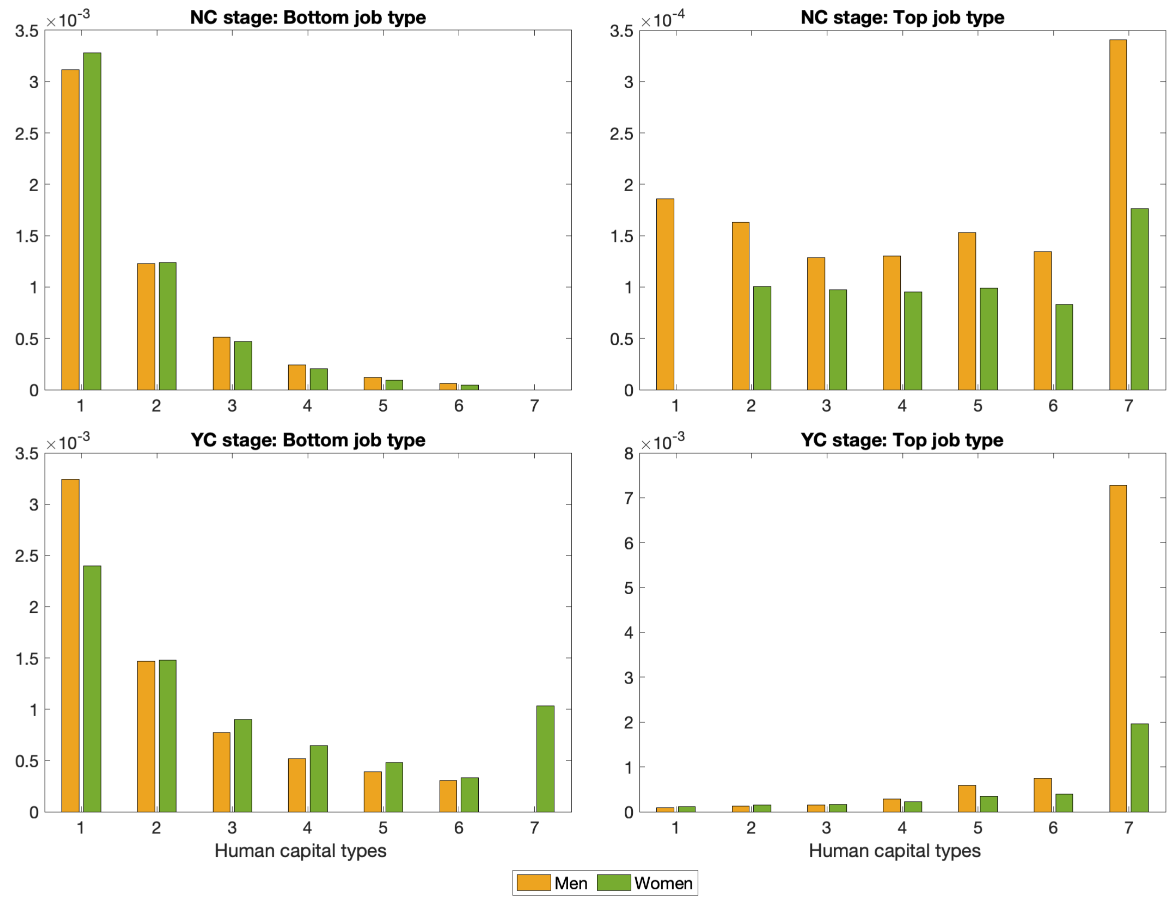
The estimates imply an equilibrium allocation where the most productive jobs (category 7) are less likely to match with women in the "no child" stage. As shown in Figure 9, jobs in the top category do not match with women of the lowest HC type, whereas the same jobs do match with low-HC men. This pattern holds true regardless of workers' amenity preferences. Such differential job allocations against women in early career could have long-term consequences given the different rates of skill accumulation across high-

[42] Although using different models and data samples, many studies that estimate CES production function also find the complementarity parameter to be negative, ranging from -0.895 in Lise, Meghir and Robin (2016) to -7.3 in Bagger and Lentz (2018). My σ estimate of -14.49 is larger in magnitude, likely because I focus on a sample of highly educated workers, whose production function may exhibit stronger complementarity compared to the general population.

and low-productivity jobs.

In the "young child" stage, men of the highest HC type do not match with bottom jobs, whereas high-HC women are willing to take the low-productivity jobs in YC stage (see bottom left panel of Figure 9). This is because high-HC men have great outside options and a high likelihood of matching with top jobs, so they would rather wait in unemployment than take a low-end job. In contrast, high-HC women have a lower reservation value than their male counterparts because women are subject to high separation rates in YC, so there is less value in waiting for better jobs to arrive.

FIGURE 9. Sorting of men and women in bottom vs. top jobs



NOTES: The bars represent the equilibrium sorting of men and women across jobs as implied by the parameter estimates. Each bar is the total measure of workers of a given HC type in a bottom job (productivity category 1) or in a top job (productivity 7). Both bottom and top jobs are of regular amenity provision type.

In the "non-fecund" stage where workers have moved beyond child-rearing ages, match formation decisions are similar for men and women.

The preference estimates suggest that men and women have similar valuations for amenities before having children, but women's preference increases to almost twice as much after childbirth. However, women's switch into high-amenity jobs are not as pronounced and sudden in the model as in the data. This is because in a frictional environment in the model, opportunities to move to high-amenity jobs may not arise immediately after childbirth. Anticipating the rate of encountering high-amenity jobs, some women already sort into these jobs before childbirth and others gradually move into them after having children.

The model fits the life-cycle wage profiles of men and women very well, and is able to replicate key moments of the data. Figure A3 summarizes the fit of the model moments compared to targeted data moments. Men have higher wages than women throughout the life-cycle, are less represented in low-productivity jobs and more represented in high-productivity jobs. The proportion of women in high-amenity jobs increases after childbirth, and the gender wage gap increases in the first years after birth before coming down 10 years afterwards. All these important qualitative features of the data are captured by the model.

4.5 Alternative specification

In order to evaluate the importance of equilibrium sorting, I estimate an alternative model with no capacity constraint akin to a conventional search model.

I set vacancy values to zero for all jobs, assuming that free entry leads to zero profit for all employers (given some underlying vacancy cost parameters). In this alternative model, sorting does not occur because surplus values are always increasing in job productivity for all worker skill types. Since all workers rank jobs the same way, matching with top jobs is essentially random. Consequently, highly skilled workers do not disproportionately end up in highly productive jobs, even if the CES function were to exhibit strong complementarity.

I estimate the alternative model with the same targeted data moments. Since job allocations are no longer endogenous in this model, I fix the transition parameters Λ and estimate only the core parameters Θ. The parameter estimates are presented in Appendix Table A4. Notably, the production function estimates imply that worker skills and job productivity are weak complements ($\sigma = 0.54$) in the alternative model, as opposed to strong complements in the baseline model. In order to match the increasing wages over the life-cycle without any sorting mechanism, the alternative model attributes the observed wage growth in the data to a higher rate of human capital accumulation (baseline HC growth rate $d_1 = 0.023$ in the alternative model compared with 0.001 in the baseline model).

Unsurprisingly, the alternative model fails to replicate the sorting patterns of men and women over the life-cycle although it manages to fit the wage profiles. As workers advance to better jobs, the shares of men and women working in job type 1 (the least productive category) decline significantly over the life-cycle in the data, and the shares in a highly productive job (category 5) increase considerably (Appendix Figure A4). However, these shares remain relatively flat over time in the alternative model.

As workers gain human capital over the course of their careers, wages grow because the skills they accumulate would contribution to production and lead to higher surplus values. However, in this alternative model, higher levels of human capital do not make the experienced workers move from low- to high-productivity jobs any more than the low-skilled workers. In other words, the sorting profiles of both men and women are flat because the model without capacity constraint does not produce positive assortative matching. Moreover, the difference between the shares of men and women in a high-productivity job (category 5) is too small in this model, and the gender gap does not widen nearly as much as in the data. This is because top jobs match with both men and women of all types in the alternative model, and it is unable to generate a "glass ceiling" for women in top positions.

5 Gender gap decomposition and policy counterfactuals

Given model estimates, I first decompose the life-cycle gender gaps by sequentially shutting off additional channels. Then I compare three policies aimed at reducing gender inequality: (1) more parental leave months earmarked for fathers; (2) equal hiring at top jobs; and (3) equal pay for equal work.

5.1 Decomposition of the life-cycle gender wage gap

I first decompose the gender gaps in hourly log wages over the life-cycle. There is no straightforward way of decomposing the gender wage gap, since the channels interact with each other and the sequence of the exercises might matter. In the following decomposition, I focus on the impact of child-related career interruptions on human capital accumulation and its interactions with employers' decisions, while considering preference for amenities separately. Figure 10 shows the cumulative effects on the gender wage gaps when additional channels are implemented (see Table A5 for corresponding numbers). Table A6 conducts a different counterfactual exercise that shows the individual effect of one channel at a time (while holding other channels fixed), and illustrate any interactions between the mechanisms.

The top black solid line in Figure 10 shows the gender wage gap implied by model estimates. The model is able to replicate both the level and the hump-shaped pattern of the log wage gap in the data. Average wage gaps in three-year windows increase from 9.1 log points in the first three years to 15.5 log points in mid-career, and then declines to 13.2 log points later in the life-cycle (Table A5). The narrowing of the gender wage gap fifteen years after labor market entry is mainly driven by men already reaching a high point on the job ladder so they have little room for advancement, while women are returning from child-related career break and are starting to catch up.

The life-cycle gender wage gap is decomposed in three steps. First, I allow men and women to have the same child-related interruptions, while keeping equilibrium wages and job allocation decisions fixed. The red and yellow dotted lines show the resulting wage gaps after (1) equalizing parental leave duration, and (2) equalizing separation rates in addition to parental leave, respectively.[43] Since equilibrium effects are not considered at this point, the wage changes after equalizing parental leave can be interpreted as direct effects of experience gains (losses) of women (men). Greater job experience not only leads to higher wages in the same job, but also qualifies the worker for better positions on the ladder due to PAM, further boosting up wages for women (and lowering those for men). Equalizing separation rates can have two direct effects. First, women stay longer on the job with a reduced separation rate, and gain more human capital that leads to higher wages. Second, women fall off the job ladder less often, so they can now extract more rents through re-negotiations and on-the-job search, and advance more on the career ladder.

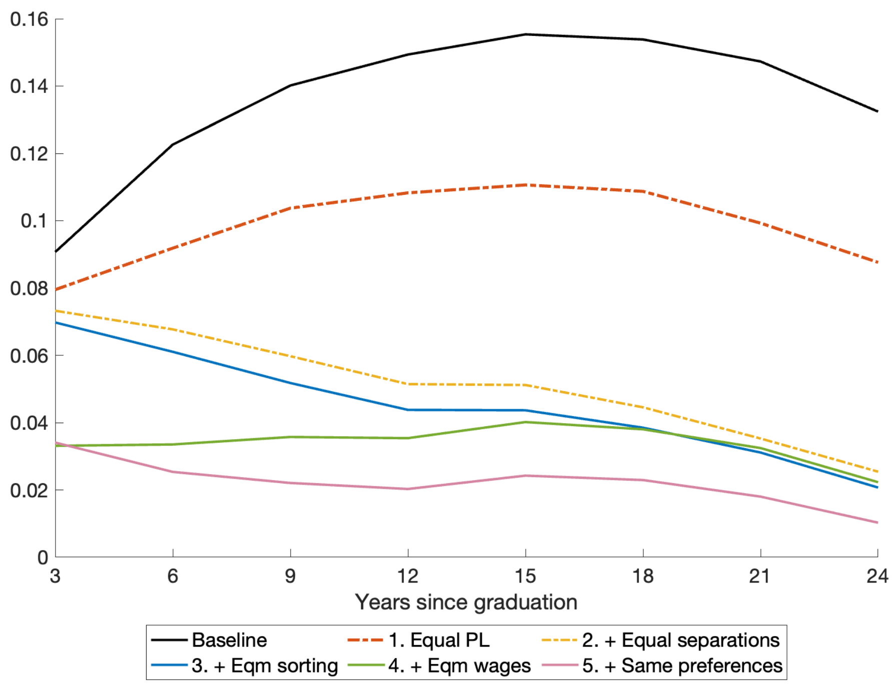
Parental leave and job separations channels both have large direct effects on the gender wage gap, particularly in later stages of a career. These channels together lead to only a 1.7 log point reduction in the wage gap during the early career phase (years 1 to 3), as educated men and women behave similarly before having children. However, the direct effects of changes in labor force attachment accumulate over time. By late career (16 to 24 years post-graduation), these channels contribute to a substantial portion of the wage gap, accounting for 11 out of 15 log points.

The second step of the decomposition is to measure the effects of child-related inter-

[43] Instead of women taking 18 months and men 2 months, they each take 10 months in the counterfactual so the total number of PL months remains the same as before. The counterfactual separation rate is chosen somewhere in between the men and women's E-to-U rates, such that the total measure of employed workers is the same across the counterfactuals.

ruptions on (3) equilibrium job allocations and (4) equilibrium wages. When parental leave durations and separation rates are equalized between men and women, employers who anticipate similar behaviors of male and female workers around childbirth would adjust decisions on both hiring and wage margins.

FIGURE 10. Gender wage gap decomposition



NOTES: The lines represent the log wage gap between men and women over the life-cycle. The top black solid line is the wage gap based on model estimates. The colored lines are the counterfactual wage gaps after adding one channel at a time: (1) Equal PL duration by gender, without changing equilibrium wages and job allocations. (2) Add equal separation rates, without equilibrium effects. (3) Implement the new equilibrium **job allocations** implied by equal PL and separations. (4) Implement new equilibrium **wages**. (5) Same preference for amenities by gender in addition to (1)-(4).

In order to measure changes in equilibrium job allocations, I allow match formation and mobility to change to the new equilibrium while keeping wage policies the same as in the old equilibrium. In the new equilibrium, jobs in the highest productivity category that did not hire low-HC women now start matching with both men and women in NC stage. High-HC men who did not accept low-end jobs in YC stage now start taking them. Even though match formation decisions only change for a handful of types of workers and firms, changes in job allocations would propagate to the rest of the distribution. Having more women in top jobs implies that some men would be "pushed" to lower jobs. Vice versa, more men being drawn to bottom jobs means women will contact these vacancies with lower probability and encounter vacancies elsewhere with a relatively higher probability. These changes in allocations, however, have only a small impact on the overall gender wage gap (see Table A5). They lead to an additional 0.4 to 0.8 log points' reduction in the gender wage gap. The small effect might be driven by the fact that allocation changes only occur for a small group of people, who do not influence average wages considerably. Another reason might be that wages are kept to the previous equilibrium where there are still substantial wage discounts against women especially at top firms.

Next, I implement new equilibrium wages under equal PL and separations in addition to the new equilibrium job allocations. Employers' differential wage offers to men and women are responsible for a substantial portion of the wage gap in early career, accounting for 3.7 out of 9.1 log points during the first 3 years post-graduation. The equilibrium wage effects dissipate over time as more and more workers move beyond child-rearing ages, although the human capital effects from earlier job allocations are carried over to infertile ages.

In the third step, I compute a new equilibrium based on equal valuations of family-friendly amenities between men and women,[44] in addition to equal parental leave and separation rates. There are both wage and mobility changes in the new equilibrium, and altogether these changes explain an additional 1.5 to 1.2 log points of the gender wage gap in mid- and late-career, respectively. Since men and women have very similar values for amenities in the "no child" stage, preference for job amenities explains little of the wage gap in early career.

Table A8 conducts a different counterfactual. It shows the effect of one channel at a time, while keeping the other channels fixed to the old regime.[45] Notably, there are intriguing interactions between the channels, as the combined effect of multiple channels often differs from the sum of their individual impacts. For example, when equal parental leave and separation rates are combined with the new equilibrium job allocations (while maintaining the previous wage policies), the joint effect on the wage gap is more pronounced. This is because the advantages of increased labor force attachment for women are enhanced by the greater availability of higher-level job opportunities.

It may seem counterintuitive that the combined effect with new equilibrium wages, in contrast, is smaller in magnitude than the sum of the individual impacts. This occurs because changes in equilibrium wage offers vary across different job types and life stages. As we transition from a system with unequal parental leave and E-to-U rates to one with

[44] I set women's preference equal to men's before childbirth, and do now allow their preferences to change after childbirth.

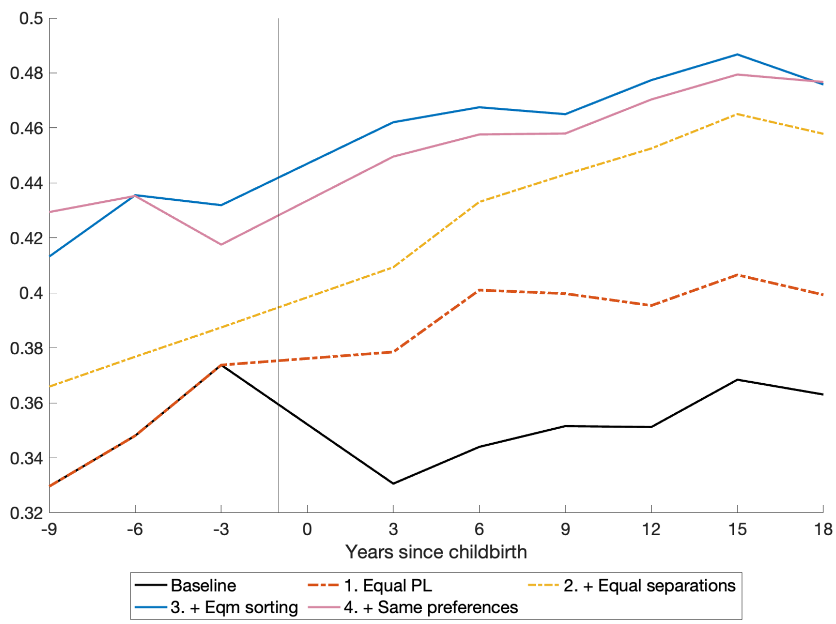
[45] For example, the row "(4) Equilibrium wages" shows the effects of new equilibrium wages on the gender wage gap, while fixing PL and separation parameters to the old (unequal) rates, and keeping equilibrium allocations to the old equilibrium.

equal conditions, wage offers to women typically increase in the “No Child” stage but may decrease in some worker-job matches in the “Young Child” stage. This happens because women are willing to accept lower wages in some positions in return for the improved job prospects in YC stage in the new equilibrium.

5.2 Under-representation of women in top positions

While wages are a key focus in discussions about gender inequality, another important question is why so few women reach top-level positions compared to men. To what extent is the “glass ceiling” shaped by individual choices regarding labor supply and job amenities, versus firms’ decisions regarding job allocations for men and women?

FIGURE 11. Counterfactual female shares in top jobs around childbirth



NOTES: The lines represent the share of women in top jobs (top two job productivity categories), by years since first childbirth. The bottom black solid line is the female share implied by model estimates. The colored lines are counterfactual female shares when additional channels are added sequentially: (1) Equal PL duration by gender, without equilibrium effects. (2) Add equal separation rates by gender, without equilibrium effects. (3) Implement the new equilibrium allocations implied by equal PL and separations. (4) Same preference for amenities by gender in addition to (1)-(3).

Figure 11 addresses the question by investigating the share of women in the most productive jobs, which are jobs in categories 6 and 7 in the model that correspond to mostly management and professional positions in the data. In the estimated model, women’s representation in top positions rises slowly from 33 to 37% in the nine years leading up to childbirth. However, this percentage declines immediately after the birth of the first child, as depicted by the bottom black solid line of Figure 11 (see also the related table

Table A7).

Similar to the decomposition in subsection 5.1, I proceed in 3 steps. First, I eliminate gender differences in labor force attachment without changing the equilibrium job allocations. The red dotted line in Figure 11 illustrates that equalizing parental leave increases the proportion of women in top positions by approximately 5 percentage points over the ten years following childbirth. The region between the red and black lines reflects a direct effect of human capital gains (losses) of women (men). The model estimates imply that there are strong production complementarities between worker skills and job productivities, so the forces of positive assortative matching would push women to highly-productive jobs as their human capital improves.

Similarly, equalizing E-to-U rates also boosts the female share in top positions, by an amount equivalent to the region between the yellow and red dotted lines. With increased job experience and fewer career interruptions, women advance more into management through assortative matching and on-the-job search. Their representation in top roles increases by about 3 percentage points before childbirth and nearly 6 percentage points fifteen years after childbirth. Together, these two labor force attachment channels alone could eliminate over half of the gender imbalance in top positions following childbirth. In essence, most top positions are open to both men and women in the baseline economy, but fewer women with the requisite accumulated experience are available compared to men.

Second, I implement the new equilibrium job allocations and sorting after equalizing parental leave and separation rates. As parental leave becomes shorter and job spells more stable for women (and the opposite for men), employers and workers might make different match formation decisions. The blue solid line in Figure 11 shows the resulting female share in top positions as some jobs change their equilibrium hiring policies. In the "No Child" stage, certain top jobs that were previously inaccessible to young women in the baseline economy now begin to match with them in the new equilibrium. These highly-productive positions provide significant learning opportunities in early career stages, and women's human capital grows at an overall faster rate than in the baseline economy. Moreover, as women anticipate a less interrupted career going forward, they also shift their job search towards better positions. As a result, high-skilled women start to reject more low-end jobs, further enhancing their human capital.

Better job opportunities in the "No Child" stage also affect choices after having children. Women's improved skills and career prospects make them less likely to (endogenously) quit their jobs after parental leave, and this reduced likelihood of quitting has lasting positive effects on their career trajectories for decades. The job allocations channel primarily improves women's representation in top jobs in the period before childbirth, increasing it by 4 to 6 percentage points over the ten years prior to birth. Even though employers' job allocation decisions do not change in the non-fecund stage, the initial access to top jobs have persistent effects in the long run – it increases women's representation in top managerial jobs by 4 to 2 percentage points five to fifteen years after childbirth.

Third, I compute the new equilibrium implied by equal amenity preferences across genders. As women's valuation for family-friendly amenities is reduced to men's level, it moderately decreases the proportion of women in top positions by 1 percentage point both before and after childbirth. This is because jobs in the top categories that provide flexibility and other amenities now become less attractive to women, and some women in these jobs might switch to lower-ranked jobs that provide less amenities but pay more. Therefore, equalizing men and women's preferences slightly exacerbates women's underrepresentation in top jobs, but improves the gender wage gap (as shown in Figure 10).

5.3 Counterfactual policy experiments

The section below considers three counterfactual policies that aim to reduce gender gaps in the labor market – a "daddy month" parental leave expansion, equal hiring in top jobs, and an equal pay policy. I compute the new equilibrium and quantify the effects of each policy on the gender wage gap as well as the female share in top positions over the life-cycle.

5.3.1 Daddy months

In Finland and many other Nordic countries, there is generous wage-replaced parental leave of durations from 6 months to over a year. Leave days could be split evenly between the parents, but it is almost always the mother who takes up most if not all of the shared leave. Many of these countries have then introduced 1 to 3 months of "daddy months" to encourage fathers to spend more time with the baby (Dahl, Løken and Mogstad, 2014).

Finland also implemented a family leave reform in 2022 that incentivized parents to share leave more equally. Under the new policy, each parent is allowed 6 months of paid leave, out of which 4 months are non-transferable (compared with 2 non-transferable months in the old regime).[46] Therefore, I consider a counterfactual where fathers' leave

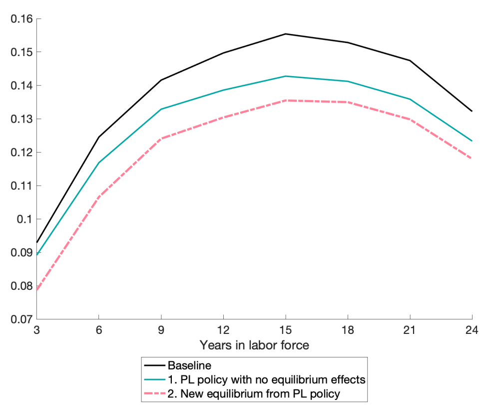
[46]It is too early to assess the effects of the reform on actual leave take-up. However, since the total paid leave duration and home care allowances remain unchanged, I assume the same in the model counterfactual. I also assume that the earmarked leave for fathers is binding under the new regime, as it was in the

increases by 2 months per child while mother's parental leave decreases by 2 months. To do this, I calibrate the parental leave exit shocks η_m and η_f so that men's leave duration per child increases from 2 to 4 months on average, while that of women's decreases from 18 to 16 months.

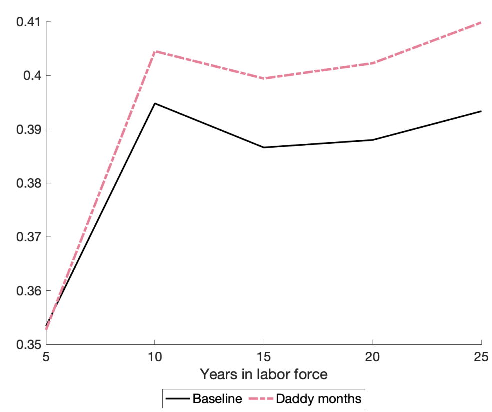
The daddy month reform is quite effective in reducing the gender wage gap throughout the life-cycle. As shown in Figure 12(A), the wage gap closes by 15% during the first 3 years of working, and over 10% afterwards. About half of the impact on wages comes from a change in equilibrium wage offers during pre-child years. Even though the shift of two months is not enough to change the job allocation decisions in years prior to childbirth, women's wages are now closer to men's when they are hired. Women also gain more human capital during mid-career because they return to work sooner after having children, while men accumulate less. This slightly balances the gender ratio in top jobs as the proportion of women increases from 39 to 41 percent by year 25 (see Figure 12(B)).

FIGURE 12. Counterfactuals under daddy months policy

(A) Gender wage gap



(B) Female proportion at top jobs



One caveat of this policy is that it might not result in a pareto improvement – the progress in women's careers might come at the expense of men's. In order to assess the overall social value of the policy, define social welfare (SW) as the sum of the production of the employed matches and the home production of the unemployed net of the total cost of vacancies:

$$SW = \sum_{g,t,\boldsymbol{x}} bk \cdot u_t^g(\boldsymbol{x}) + \sum_{g,t,\boldsymbol{x},\boldsymbol{y}} f(k,p) \cdot h_t^g(\boldsymbol{x},\boldsymbol{y}) - \sum_{\boldsymbol{y}} c \cdot v(\boldsymbol{y}).$$

By the time men become fathers, they are already in slightly more advanced positions than women and are producing more output, so the output loss of having men spend

previous regime.

2 months at home cannot be fully compensated by output gains of women working 2 months more. However, the net loss in social welfare is very small (only 0.02% of total welfare). Also, paying men on parental leave is more costly since the benefits are proportional to wages and men typically earn more than women. In order to fund the new policy, the tax rate on flow output increases modestly from 2.80% to 2.88%.

5.3.2 Equal hiring policy in top jobs

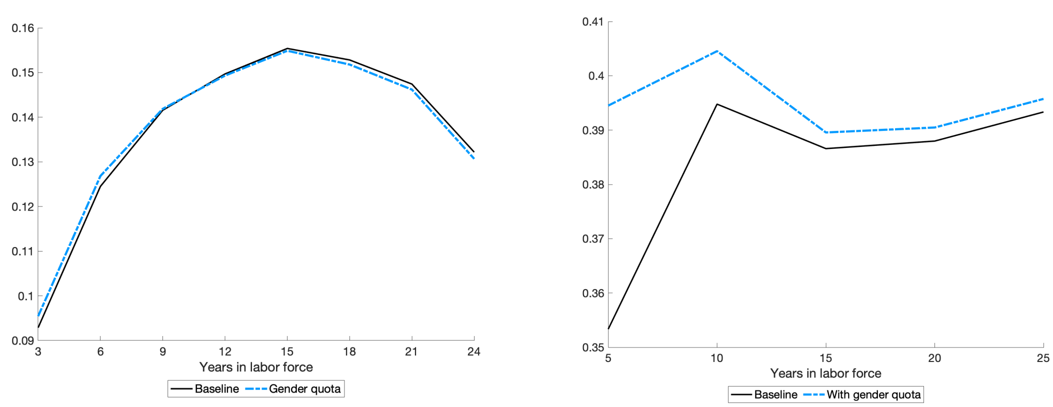
To address the under-representation of women in top-earning jobs, many countries have passed legislature to require a certain percentage of female board members in public companies. Finland requires state-owned enterprises to reserve 40% of board seats to female directors. However, the evidence on the effectiveness of these policies in reducing gender gaps is mixed at best (Bertrand, Black, Jensen and Lleras-Muney, 2018).

There is no direct way of implementing a gender quota in the model since the proportion of women in a particular job category depends not only on the optimal hiring rule of the job, but also on the transition rates and workers' mobility to all other jobs in equilibrium. In practice, I implement an "equal hiring" policy that requires the top jobs (those in the highest productivity category) to have the same hiring rule towards a woman and a man of the same $\boldsymbol{x}$ type.

The policy essentially changes hiring rules of top positions towards low-HC women in the "no child" stage. Since these matches would not have been formed in the absence of the equal hiring policy, there is no standard wage protocol about how to split the (negative) match surplus. In this exercise, I assume that the employer sets the wage to cover the vacancy value of the job, and the worker gets the rest of the match value.

FIGURE 13. Counterfactuals under equal hiring policy

(A) Gender wage gap (B) Female proportion at top jobs



Unsurprisingly, banning hiring discrimination at top jobs improves women's representation in those jobs during the early years of workers' professional lives. Figure 13(B) shows that the female share increases from 35.5 to 39.5 percent in top jobs during the first 5 years of work. However, this effect is very short-lived. Since the equal hiring policy does not address child-related interruptions, women start falling behind men in human capital levels soon after childbirth, and are thus less likely to stay in highly productive jobs later on due to forces of PAM. The proportion female in top jobs almost falls back to baseline levels during child-rearing years. The overall effect of the policy on the share of women in top jobs is only slightly positive by the end of the life-cycle.

Even though the equal hiring policy improves women's representation at top jobs, employers undo this policy by exerting more wage discounts. Women hired under the new policy receive lower wages than men in the same job during the early years of the life-cycle. This is because employers are now required to form matches with all women even though some matches generate negative surpluses; as a result, the new female hires have to "compensate" the employers by accepting sub-par wages. Since the new hires are a small proportion of the working population, the overall wage gap only increases by a small amount (by 3% in 6 years). However, being employed in high-productivity jobs in early career allows young women to gain skills at a faster rate, and the human capital gains more than compensate for the initial wage loss. Figure 13(A) shows that the negative impact of the policy on women's wages disappears nine years after labor market entry.

There is a small welfare gain when women gain access to top jobs. This is because when employers make hiring decisions, they only care about maximizing their individual profits and do not take into account the social benefit of upgrading workers' human capital. The net welfare gain from the equal hiring policy is 0.01%.

5.3.3 Equal pay policy

Many OECD countries have passed some form of Equal Pay Act that requires men and women in the same workplace be given equal pay for equal work. The Finnish Equality Act requires companies with 30 or more full-time employees to draft a gender equality plan, which should include an assessment of pay differences between men and women who perform work of equal value.[47]

In the equal pay counterfactual, I require men and women of the same $\boldsymbol{x}$ type working

[47] Details of the Equality Act and related reforms can be found at: https://www.finlex.fi/en/laki/kaannokset/1986/en19860609

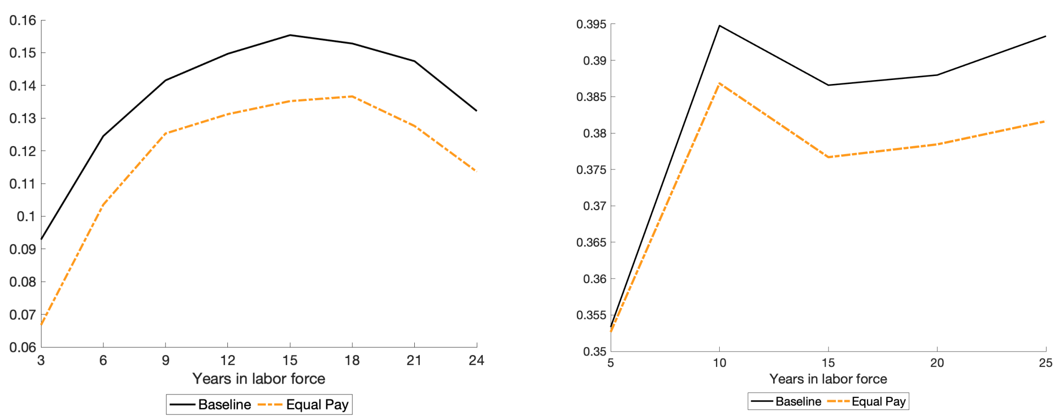
in the same $\boldsymbol{y}$ job to receive the same starting wage. I compute the equivalent lifetime value of the female worker $W_{0t}^{f}(\boldsymbol{x},\boldsymbol{y})$ implied by having men's wages $\omega_{0,t}^{m}$ in each age segment t, and re-calculate employer's share in the surplus:

$$\underbrace{\Pi_t^f(\omega_{0,t}^m,\boldsymbol{x},\boldsymbol{y})-\Pi_0(\boldsymbol{y})}_{\text{employer's share}}=S_t^f(\boldsymbol{x},\boldsymbol{y})-\underbrace{\left(W_t^f(\omega_{0,t}^m,\boldsymbol{x},\boldsymbol{y})-U_t^f(\boldsymbol{x})\right)}_{\text{worker's share}}.$$

When the worker's value W_{0t}^{f} is required to increase, the employer's portion might become negative, in which case the match would dissolve.

FIGURE 14. Counterfactuals under equal pay policy

(A) Gender wage gap (B) Female proportion at top jobs



I simulate the workers' careers with the equal wage policy, allowing matches to dissolve if the employer's value $\Pi_t^g(\omega_{0,t}^m,\boldsymbol{x},\boldsymbol{y})$ fall below the vacancy value $\Pi_0(\boldsymbol{y})$. Figure 14 shows that the equal pay policy unsurprisingly reduces the gender wage gap by 2 to 4 log points. However, some matches are no longer sustained in the periods after having children. As a result, women are more likely to be unemployed and fall off the career ladder, although the effect size is very small. Figure 14(B) shows that the proportion of women in top jobs decreases by 1 percentage points in years 15-25.

6 Conclusion

This paper studies the mechanisms underlying gender gaps over the life-cycle — workers' human capital accumulation, preference for amenities, and employers' wage-setting and job allocation decisions. I propose an equilibrium search model with capacity constraints, production complementarities, fertility and parental leave, and taste for job amenities. The model is estimated using matched employer-employee data from Finland combined with occupation-level data on amenities from the Finnish Quality of Work Life

Survey.

Men and women behave very differently in the labor market especially after having children. Employers take into account these gender differences and allocate different jobs to men and women even before they have children. The model estimates imply that a large portion of the gender wage gap in early career can be attributed to employers' differential sorting and wage-setting by gender based on fertility-related concerns. In late career stages, gender disparities in labor force attachment account for the majority of the wage gap.

The most effective policies for reducing gender gaps are those that alleviate women's family responsibilities, such as expanding childcare options to lower women's separation rates and increasing parental leave for fathers. These measures would not only help women gain more on-the-job experience but also shift firms' expectations, leading employers to reduce wage markdowns and improve job opportunities for women. On the other hand, policies aimed at eliminating hiring discrimination in top jobs might reduce women's wages in early-career, while equal pay policies can lower the proportion of women in top positions as employers adjust their hiring strategies.

Overall, the policy counterfactuals suggest that it might be difficult to achieve gender equality in the workplace without greater equality in family responsibilities, given the sizable effects of employers' equilibrium decisions. Requiring equality in one margin (either wages or job allocations) often prompts firms to counteract the policy on the other margin. Requiring equality across both margins could be challenging and costly to enforce, as forward-looking, rational employers have an inherent incentive to deviate from such mandates.

An extension of the model might involve formalizing intra-household decisions, where spouses jointly choose their parental leave lengths and separation rates while taking into account their labor market prospects. Employers' priors that women are more prone to career interruptions might become a self-fulfilling prophecy if the resulting wage and job opportunity discrimination leads women to specialize in household production. Conversely, policies that promote gender equality in the labor market may trigger considerable feedback effects if households respond by sharing family responsibilities more equally. Therefore, increased equality in the labor market might reinforce gender equality within the household, and vice versa. Quantifying the long-run impacts of such propagating effects is left for future research.

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Appendix

Appendix A Tables and figures

TABLE A1. Principal component analysis for the amenity index

Variables	Factor loading	Unexplained proportion
1. Work from home	-0.315	.522
2. Flexible start/end	-0.378	.312
3. Flexible hours	-0.256	.683
4. Run errands during work	-0.338	.450
5. Overtime without pay	0.369	.344
6. Contacted after work	0.403	.217
7. Too much overtime	0.375	.322
8. Actual hours worked (LFS)	0.347	.419
9. Proportion part-time	0.140	.905

NOTES: The table shows the factor loading of each variable for the first principal component. Negative amenities (variables 5 to 8) are multiplied by -1 before entering the principal component analysis, so all the variables can be interpreted as good amenities.

TABLE A2. Summary statistics by job productivity types

Job productivity types	1	2	3	4	5	6	7
Number of workers	27,192	37,155	38,003	41,466	37,309	22,161	13,136
Number of workers per job	2.00	4.20	4.03	4.05	4.04	2.91	2.24
Mean log-wages	2.64	2.96	3.10	3.24	3.39	3.55	3.83
SD of log-wages	0.212	0.043	0.041	0.041	0.044	0.056	0.133
p	1	1.37	1.58	1.82	2.10	2.49	3.28
% **Clerical jobs**	33.51%	7.37%	4.49%	2.91%	1.47%	1.01%	0.70%
% **Associates**	23.03%	18.19%	28.42%	19.54%	13.02%	9.50%	3.46%
% **Professionals**	42.01%	72.26%	63.6%	70.03%	70.89%	59.97%	35.27%
% **Managers**	1.45%	2.17%	3.49%	7.52%	14.62%	29.52%	60.56%

TABLE A3. Calibrated Parameters

Parameters		Estimate	Moments
Fertility rate	χ	0.0104	On average 1.7 children
Ageing rate	γ	0.0042	95% of births happen before age 45
PL ending rate	η^m	0.9524	Parental leave duration of men
	η^f	0.0278	Parental leave duration of women
Production ratio in PL	R	0.7	Ginja, Karimi and Xiao (2023)
Vacancy cost	c	1.6	Lise, Meghir and Robin (2016)
Job distribution	$\varphi(y, \alpha)$	Figure 7	K-means clustering by long-term average wage within firm-occupation

TABLE A4. Parameter estimates of alternative model

$\boldsymbol{\Theta}$ parameters		Estimates	S.E.
Complementarity	σ	0.540	0.384
Relative productivity	a	0.855	0.063
TFP	A	13.026	4.614
Baseline HC rate	d_1	0.023	0.001
Proportional HC rate	d_2	0.000	0.015
Men's value for amenities	μ_m	0.163	0.062
Women's value for amenities	μ_f	0.174	0.093
Preference increase in motherhood	M	0.425	0.079
Worker's bargaining	β	0.665	0.192
Home productivity	b	5.437	1.791
Initial distribution - men	ξ_m	2.070	1.071
Initial distribution - women	ξ_f	2.650	1.286

TABLE A5. Counterfactual gender gaps in log hourly wages corresponding to Figure 10

	Years since graduation							
	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24
Baseline gap (log points)	**9.07**	**12.26**	**14.02**	**14.94**	**15.54**	**15.38**	**14.73**	**13.24**
Resulting wage gaps:								
(1) Equal parental leave	7.95	9.18	10.37	10.83	11.06	10.87	9.93	8.76
(2) + Equal separations	7.32	6.77	5.97	5.14	5.12	4.45	3.53	2.54
(3) + Equilibrium allocations	6.98	6.10	5.18	4.38	4.37	3.85	3.11	2.07
(4) + Equilibrium wages	3.31	3.35	3.57	3.54	4.02	3.80	3.24	2.23
(5) + Same preferences	3.41	2.54	2.21	2.03	2.42	2.29	1.80	1.03
Marginal changes:								
(1) - baseline	-1.12	-3.08	-3.64	-4.11	-4.48	-4.51	-4.80	-4.48
(2) - (1)	-0.63	-2.41	-4.40	-5.68	-5.94	-6.42	-6.40	-6.22
(3) - (2)	-0.35	-0.67	-0.80	-0.77	-0.75	-0.61	-0.42	-0.47
(4) - (3)	-3.66	-2.75	-1.60	-0.84	-0.35	-0.05	0.13	0.16
(5) - (4)	0.10	-0.82	-1.37	-1.51	-1.59	-1.51	-1.44	-1.20

TABLE A6. Effect of each channel on gender wage gap and their interactions (log points)

	Years since graduation							
	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24
Baseline gap (log points)	**9.07**	**12.26**	**14.02**	**14.94**	**15.54**	**15.38**	**14.73**	**13.24**
A. Individual effect of channel:								
(1) Equal parental leave	-1.12	-3.08	-3.64	-4.11	-4.47	-4.51	-4.80	-4.48
(2) Equal separations	-0.71	-2.54	-3.99	-5.05	-5.73	-6.06	-5.94	-5.91
(3) Equilibrium allocations	-0.51	-0.92	-0.97	-0.78	-0.62	-0.50	-0.56	-0.55
(4) Equilibrium wages	-4.17	-3.71	-2.75	-1.91	-1.23	-0.85	-0.70	-0.72
(5) Same preferences	0.03	-0.64	-1.05	-1.34	-1.45	-1.62	-1.55	-1.43
B. Interactions:								
Add up (1) and (2)	-1.83	-5.62	-7.64	-9.16	-10.21	-10.57	-10.74	-10.39
Joint (1) and (2)	**-1.74**	**-5.49**	**-8.04**	**-9.79**	**-10.42**	**-10.93**	**-11.20**	**-10.70**
Add up (1) to (3)	-2.34	-6.54	-8.61	-9.93	-10.82	-11.06	-11.30	-10.95
Joint (1) to (3)	**-2.09**	**-6.16**	**-8.84**	**-10.56**	**-11.17**	**-11.53**	**-11.62**	**-11.17**
Add up (1) to (4)	-6.51	-10.25	-11.36	-11.84	-12.05	-11.92	-12.00	-11.67
Joint (1) to (4)	**-5.76**	**-8.91**	**-10.44**	**-11.40**	**-11.52**	**-11.58**	**-11.49**	**-11.01**
Add up (1) to (5)	-6.47	-10.89	-12.41	-13.18	-13.50	-13.54	-13.55	-13.11
Joint (1) to (5)	**-5.66**	**-9.72**	**-11.81**	**-12.91**	**-13.11**	**-13.09**	**-12.93**	**-12.21**

TABLE A7. Counterfactual female shares in top jobs corresponding to Figure 11

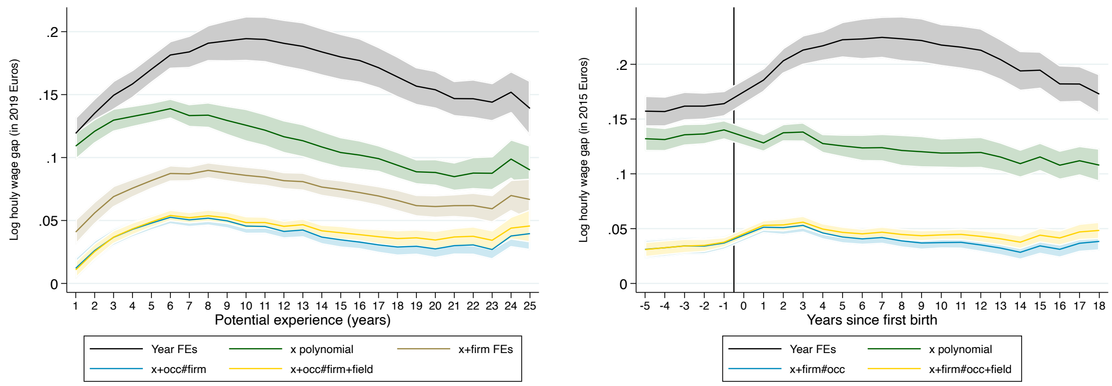
	Years since first childbirth								
	-9 to -7	-6 to -4	-3 to -1	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18
Baseline female share (%)	**33.0**	**34.8**	**37.4**	**33.1**	**34.4**	**35.2**	**35.1**	**36.8**	**36.3**
Female shares after:									
(1) Equal parental leave	33.0	34.8	37.4	37.9	40.1	40.0	39.5	40.7	39.9
(2) + Equal separations	36.6	37.7	38.7	40.9	43.3	44.3	45.3	46.5	45.8
(3) + Equilibrium allocations	41.3	43.6	43.2	46.2	46.8	46.5	47.7	48.7	47.6
(4) + Same preferences	42.9	43.5	41.8	45.0	45.8	45.8	47.0	47.9	47.7
Marginal changes:									
(1) - baseline	0.0	0.0	0.0	4.8	5.7	4.8	4.4	3.8	3.6
(2) - (1)	3.6	2.9	1.4	3.1	3.2	4.3	5.7	5.8	5.9
(3) - (2)	4.7	5.9	4.4	5.3	3.4	2.2	2.5	2.2	1.8
(4) - (3)	1.6	0.0	-1.4	-1.3	-1.0	-0.7	-0.7	-0.7	0.1

TABLE A8. Effect of each channel on female shares in top jobs and their interactions (%)

	Years since first childbirth								
	-9 to -7	-6 to -4	-3 to -1	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18
Baseline female share (%)	**33.0**	**34.8**	**37.4**	**33.1**	**34.4**	**35.2**	**35.1**	**36.8**	**36.3**
A. Indiv. effect of channel:									
(1) Equal parental leave	0.0	0.0	0.0	4.8	5.7	4.8	4.4	3.8	3.6
(2) Equal separations	3.6	2.9	1.4	3.4	3.9	4.2	4.5	4.1	4.5
(3) Equilibrium allocations	4.5	4.6	3.8	4.4	4.2	2.9	3.2	2.2	1.6
(4) Same preferences	-2.5	-1.4	-0.6	-0.2	-0.4	-0.2	-0.2	0.0	0.0
B. Interactions:									
Add up (1) to (3)	8.1	7.4	5.1	12.5	13.8	11.9	12.1	10.1	9.7
Joint (1) to (3)	**8.4**	**8.7**	**5.8**	**13.1**	**12.4**	**11.3**	**12.6**	**11.8**	**11.3**
Add up (1) to (4)	5.7	6.0	4.6	12.4	13.4	11.7	11.9	10.1	9.7
Joint (1) to (4)	**10.0**	**8.7**	**4.4**	**11.9**	**11.4**	**10.6**	**11.9**	**11.1**	**11.4**

FIGURE A1. Gender wage gap decomposition

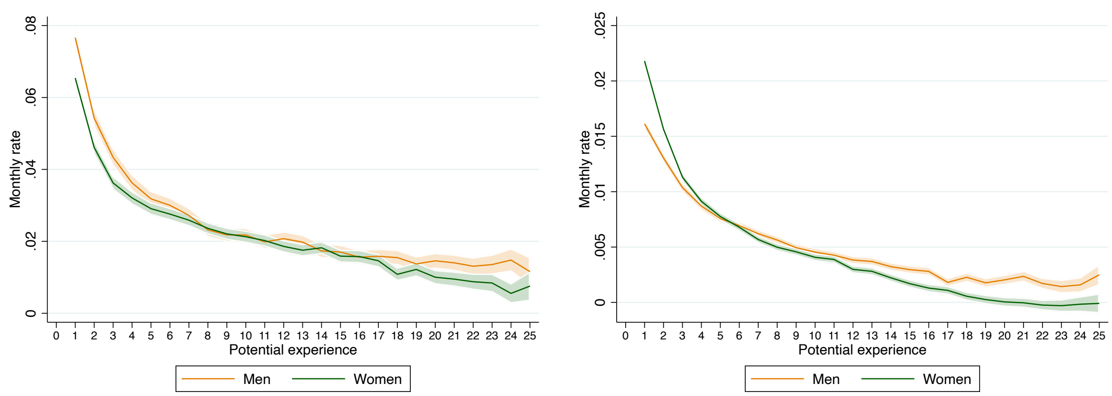
(A) Over the life-cycle (B) Around first childbirth



NOTES: The lines represent the coefficients on the male dummy interacted with potential experience or years since childbirth. Shaded areas represent 95% confidence intervals. Standard errors are clustered at the firm level. The coefficients are obtained from regressions of real log hourly wages on: (i) year dummies; (ii) a quadratic in actual experience (x) in addition to (i); (iii) firm fixed effects in addition to (i) and (ii) (I forgot to do this for around childbirth); (iv) a full set of interactions of firm and occupation dummies in addition to (i) and (ii); and (v) 3-digit field of study dummies in addition to (i), (ii) and (iv).

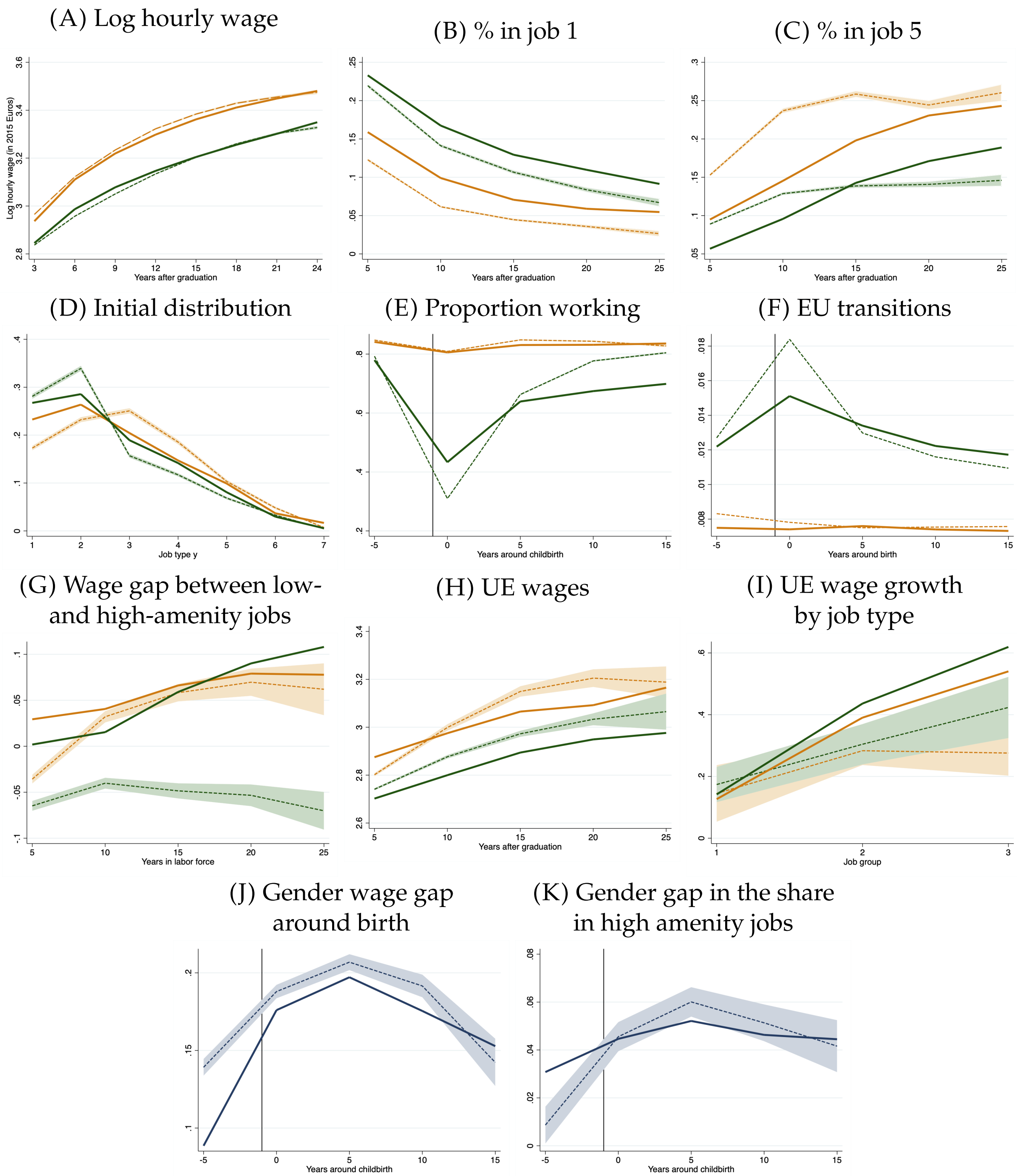
FIGURE A2. Transitions over the life-cycle

(A) U-to-E transition rate (B) Employer-to-employer transition rate



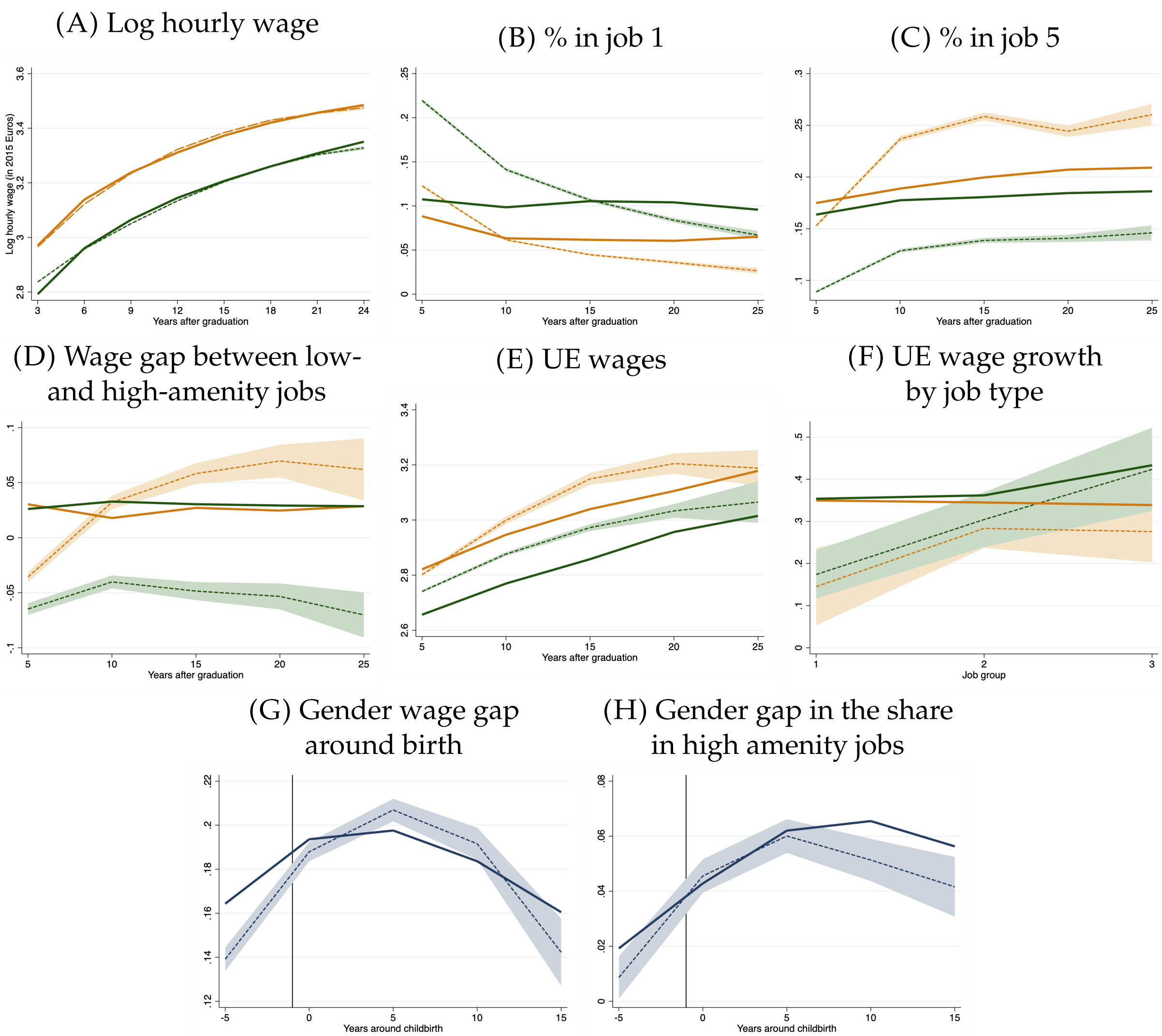
NOTES: The lines represent the coefficients obtained from regressions of outcome variables on potential experience, separately for men and women. Shaded areas represent 95% confidence intervals.

FIGURE A3. Model fit



NOTES: The solid lines represent model moments implied by parameter estimates; the dashed lines are data moments. Green denotes women, and orange denotes men. Shaded areas correspond to bootstrap standard errors from 5000 random draws of the data.

FIGURE A4. Fit of alternative model



NOTES: The solid lines represent model moments implied by parameter estimates; the dashed lines are data moments. Green denotes women, and orange denotes men. Shaded areas correspond to bootstrap standard errors from 5000 random draws of the data.

Appendix B Data description and sample selection

The Finnish Longitudinal Employer-Employee Data (FOLK) is assembled by Statistics Finland from numerous administrative registers, and covers the entire resident population aged 15 to 70 between years 1988 and 2016. FOLK provides detailed employment histories for each worker. Using the start and end dates of each employment relationship, I create a monthly employment status for each worker – employed, unemployed, or on parental leave. Since FOLK can be linked to the official population register, I also observe the birth date of each child of the worker and use it to infer the worker's parental leave status when he/she starts collecting benefits around that date.

The hourly wage data comes from the Structure of Earnings Statistics (SES). The SES consists of large-scale surveys collected by the Employers' Association in the last quarter of each year from 1995 to 2013. It covers all public sector workers and 55 to 75 percent of private sector workers depending on the year. The following groups in the private sector are either entirely excluded or at least severely under-represented: 1. small (less than 5 persons) enterprises; 2. the vast majority of non-organized (mainly small) enterprises; 3. agriculture, forestry and fisheries; 4. international organizations; 5. company management and owners and their family members; 6. the employment relationships beginning or ending during the reference month.

SES observations are on the yearly level (as opposed to daily in FOLK), and some firms might not be surveyed by SES in certain years. In the estimation, I use sample weights in the simulations to account for potential missing data from small firms.

I drop workers whose age is in the bottom or top 5 percentiles of the age distribution at graduation, so that workers in my sample are aged between 24 and 31 when they graduated master's. I drop small firms that have never had more than 2 workers during the sample period.

I only include periods after the individuals have completed their master's education. Unemployment of 2 months or less is counted as the final tenure of the previous spell. Similarly, employment of 2 months or less is counted as non-employment.

Wages and occupations are observed in the SES once a year from 1995 to 2013 in the last quarter of the calendar year. If the worker has wages from more than one employer in a quarter, I keep only the wage from the "main" job – the full-time job if there is one, or the job with the most earnings if all jobs are part-time. I trim the top 0.5% of the wage distributions in each year, which tend to be very thin and cover wide ranges. After sample selection, I have an unbalanced panel of 116,781 workers, and 25,951 distinct firm-occupations over the course of 18 years.

I remove macroeconomic fluctuations in wages and transition rates by taking out year fixed effects in all moments calculations.

Appendix C Parental leave system in Finland

The Finnish maternity allowance system was first introduced in 1964. Currently, parents are entitled to wage-replaced leave for a total of 12 months, in which 4 months are reserved for mothers, 2 months for fathers, and 6 months can be shared between the spouses. In addition, parents are entitled to Child Home Care Allowances until the child turns 3 years old. Both biological and adoptive parents are entitled to parental leave on the basis of permanent residence in Finland.

The amount of parental leave benefits is a piece-wise linear function of annual earnings in the previous employment, or social benefits collected in the case of unemployment. The rate of wage replacement depends on income tiers as shown in the following table:

TABLE A9. Maternity, paternity and parental allowances pay schedule

Annual earnings (€)	Calculation formula (annual amount in €)
up to 11,942	8,358
11,943 - 37,861	0.7 x annual earnings
37,862 - 58,252	26,503 + 0.40 x (annual earnings - 37,861)
over 58,252	34,659 + 0.25 x (annual earnings - 58,252)

After the parental leave is over, parents can continue to care for the child at home and receive the Home Care Allowances (HCA). The HCA may be paid to either parent, although it is predominantly the mother who takes up the allowance. The HCA benefit amount consists of two parts – there is a fixed amount of 338.34 euros per month for one child under 3, and a means-tested amount targeted at low-income families up to 180 euros per month. In addition, there is sibling extra and municipality-based supplements. For details of HCA, please refer to Kosonen (2014).

The benefit amount of the parental leave allowance and the HCA claimed are separately reported in the FOLK data for each individual in each calendar year. This paper uses the pay schedule in Table A9 and the fixed HCA amount adjusted by inflation to infer the total number of months of parental leave taken for each worker.

Since I observe the exact amount of parental leave benefits collected around the time of childbirth, I can pinpoint the month at which the worker stops collecting benefits. If a worker is not associated with an employer and is not collecting parental leave benefits in a particular month, he/she is considered to be unemployed.[48] According to this measure, female separation rate is already a little higher than male's prior to birth, but the big difference appears right after childbirth, where women's separation spikes and remain well above men's for many years after childbirth.

[48]If someone is unemployed for only two months or less after she stops collecting parental leave benefits, I consider it as measurement error in leave duration calculations and do not count the months as unemployment. A separation is only indicated for unemployment of 3 months or more.

Appendix D Wage determination and workers' values

Recall from equation (1) that $W^g_{0t}(\boldsymbol{x},\boldsymbol{y}) = U^g_t(\boldsymbol{x}) + \beta\, S^g_t(\boldsymbol{x},\boldsymbol{y})$. Given equilibrium values U_t, S_t, the equation below illustrates an example of the worker's value W_{0t} when he/she receives a wage ω_0 out of unemployment in the "no child" stage:

$$\begin{aligned}\Big(r + H(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, W^g_{0,NC}(\boldsymbol{x},\boldsymbol{y}) = {} & \omega^g_{0,NC}(\boldsymbol{x},\boldsymbol{y}) + q^g(\epsilon,\alpha) + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\, \widetilde{W}^g_{0,NC}(\boldsymbol{x}',\boldsymbol{y}) \\ & + \delta^g_{NC}\, U^g_{NC}(\boldsymbol{x}) + \gamma\, \widetilde{W}^g_{0,NF}(\boldsymbol{x},\boldsymbol{y}) + \chi\, \widetilde{W}^g_{0,PL}(\boldsymbol{x},\boldsymbol{y}) \\ & + s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}')\, \mathbb{1}[S' \le S] \cdot \Big[W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) - W^g_{0,NC}(\boldsymbol{x},\boldsymbol{y})\Big]^+ \\ & + s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}')\, \mathbb{1}[S' > S] \cdot \Big[W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') - W^g_{0,NC}(\boldsymbol{x},\boldsymbol{y})\Big]^+\end{aligned}$$

where the total rate of HC change is denoted by $H(k,p) = \sum_{k'} \rho_e(k'|k,p)$, and $[W]^+$ denotes $\max\{W, 0\}$. When a worker's human capital changes from k to k' in the next period, the wage does not update until there is a credible outside option.

At any point in time, the match can dissolve endogenously if surplus falls below zero:

$$\widetilde{W}^g_{0,t}(\boldsymbol{x},\boldsymbol{y}) = \max\Big\{W^g_{0t}(\boldsymbol{x},\boldsymbol{y}),\ U^g_t(\boldsymbol{x})\Big\}, \qquad t = \{NC, PL, YC, NF\}$$

When workers move from job $\boldsymbol{y}$ to $\boldsymbol{y}'$, their flow wage at the new job is ω_1 such that they get a value of $W_{1t}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')$. An example in NC stage follows:

$$\begin{aligned}\Big(r + H(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') = {} & \omega^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') + q^g(\epsilon,\alpha) \\ & + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y}')\, \max\Big(\widetilde{W}^g_{1,NC}(\boldsymbol{x}',\boldsymbol{y},\boldsymbol{y}'),\ \widetilde{W}_{0,NC}(\boldsymbol{x}',\boldsymbol{y}')\Big) \\ & + \delta^g_{NC}\, U^g_{NC}(\boldsymbol{x}) + \gamma\, \widetilde{W}^g_{1,NF}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') + \chi\, \widetilde{W}^g_{1,PL}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}') \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' \le S'] \cdot \Big[W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}'',\boldsymbol{y}') - W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')\Big]^+ \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' > S'] \cdot \Big[W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}'') - W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}')\Big]^+\end{aligned}$$

Similarly, when a poaching job $(\boldsymbol{y}')$ triggers a wage renegotiation but not a job-to-job transition (when $S' < S$), the renegotiated wage ω_2 is such that:

$$\begin{aligned}\Big(r + H(k,p) + \delta^g_{NC} + \chi + \gamma\Big)\, W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) =\ & \omega^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) + q^g(\epsilon,\alpha) \\ & + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y}) \max\Big(\widetilde{W}^g_{2,NC}(\boldsymbol{x}',\boldsymbol{y}',\boldsymbol{y}),\ \widetilde{W}_{0,NC}(\boldsymbol{x}',\boldsymbol{y})\Big) \\ & + \delta^g_{NC}\, U^g_{NC}(\boldsymbol{x}) + \gamma\, \widetilde{W}^g_{2,NF}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) + \chi\, \widetilde{W}^g_{2,PL}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y}) \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' \leq S] \cdot \Big[W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}'',\boldsymbol{y}) - W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})\Big]^+ \\ & + s\lambda \sum_{\boldsymbol{y}''} v(\boldsymbol{y}'')\, \mathbb{1}[S'' > S] \cdot \Big[W^g_{1,NC}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}'') - W^g_{2,NC}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{y})\Big]^+\end{aligned}$$

Appendix E Value functions in all life stages

The values of the non-employed workers in PL, YC and NF stages are:

$$(r + \eta^g + \gamma)\, U^g_{PL}(\boldsymbol{x}) = b(x) + \eta^g\, U^g_{YC}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) \tag{13}$$

$$(r + \chi + \gamma)\, U^g_{YC}(\boldsymbol{x}) = b(x) + \chi\, U^g_{PL}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) + \sum_{\boldsymbol{y}} \lambda\, v(\boldsymbol{y})\ \beta\, [S^g_{YC}(\boldsymbol{x},\boldsymbol{y})]^+ \tag{14}$$

$$(r + \phi)\, U^g_{NF}(\boldsymbol{x}) = b(x) + \sum_{\boldsymbol{y}} \lambda\, v(\boldsymbol{y})\ \beta\, [S^g_{NF}(\boldsymbol{x},\boldsymbol{y})]^+ \tag{15}$$

The joint values of matches in "young child" and "non-fecund" stages are:

$$\begin{aligned}(r + H(k,p) + \delta^g_{YC} + \chi + \gamma)\, P^g_{YC}(\boldsymbol{x},\boldsymbol{y}) =\ & (1-\tau)\, f(k,p) + q^g_{YC}(\epsilon,\alpha) + \sum_{k'} \rho_e(k'|k,p)\, \widetilde{P}^g_{YC}(\boldsymbol{x}',\boldsymbol{y}) \\ & + \delta^g_{YC}\Big(\Pi_0(\boldsymbol{y}) + U^g_{YC}(\boldsymbol{x})\Big) + \chi\, \widetilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) + \gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y}) \\ & + \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\ \beta\, [S^g_{YC}(\boldsymbol{x},\boldsymbol{y}') - S^g_{YC}(\boldsymbol{x},\boldsymbol{y})]^+ \end{aligned} \tag{16}$$

$$\begin{aligned}(r + H(k,p) + \delta + \phi)\, P^g_{NF}(\boldsymbol{x},\boldsymbol{y}) =\ & (1-\tau)\, f(k,p) + q(\epsilon,\alpha) + \sum_{k'} \rho_e(k'|k,p)\, \widetilde{P}^g_{NF}(\boldsymbol{x}',\boldsymbol{y}) \\ & + \delta\Big(\Pi_0(\boldsymbol{y}) + U^g_{NF}(\boldsymbol{x})\Big) + \phi\, \Pi_0(\boldsymbol{y}) \\ & + \sum_{\boldsymbol{y}'} s\lambda\, v(\boldsymbol{y}')\ \beta\, [S^g_{NF}(\boldsymbol{x},\boldsymbol{y}') - S^g_{NF}(\boldsymbol{x},\boldsymbol{y})]^+ \end{aligned} \tag{17}$$

where $H(k,p)$ denotes the total rate of HC change $\sum_{k'} \rho_e(k'|k,p)$.

Appendix F Steady-state balance equations

In a stationary equilibrium, flows into and out of any worker-job match must balance. Gender subscripts are suppressed for ease of exposition. The equations are synonymous for men and women.

$$
\begin{aligned}
u_{NC}(\boldsymbol{x}) \cdot \Bigg[\underbrace{\chi+\gamma}_{\text{fertility, ageing}} &+ \underbrace{\lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \mathbb{1}\big(S_{NC}(\boldsymbol{x},\boldsymbol{y})>0\big)}_{\text{job acceptance}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_u(x'|x)}_{\text{HC changes}} \Bigg] \\
&= \underbrace{\delta_{NC} \sum_{\boldsymbol{y},S>0} h_{NC}(\boldsymbol{x},\boldsymbol{y})}_{\text{exog. job destruction}} + \underbrace{\sum_{\boldsymbol{y},S<0} h_{NC}(\boldsymbol{x},\boldsymbol{y})}_{\text{endogenous quits}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{NC}(\boldsymbol{x}')}_{\text{HC changes}} + \underbrace{\xi_0(\boldsymbol{x})}_{\text{new entrants}}
\end{aligned}
$$

$$
\begin{aligned}
u_{PL}(\boldsymbol{x}) \cdot \Big[\eta+\gamma+\sum_{\boldsymbol{x}'} \rho_u(x'|x)\Big] \\
&= \delta_{YC} \sum_{\boldsymbol{y},S>0} h_{PL}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{y},S<0} h_{PL}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{PL}(\boldsymbol{x}') + \chi\Big[u_{NC}(\boldsymbol{x}) + u_{YC}(\boldsymbol{x})\Big]
\end{aligned}
$$

$$
\begin{aligned}
u_{YC}(\boldsymbol{x}) \cdot \Big[\chi+\gamma+\lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \mathbb{1}\big(S_{YC}(\boldsymbol{x},\boldsymbol{y})>0\big) + \sum_{\boldsymbol{x}'} \rho_u(x'|x)\Big] \\
&= \delta_{YC} \sum_{\boldsymbol{y},S>0} h_{YC}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{y},S<0} h_{YC}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{YC}(\boldsymbol{x}') + \eta \cdot u_{PL}(\boldsymbol{x})
\end{aligned}
$$

$$
\begin{aligned}
u_{NF}(\boldsymbol{x}) \cdot \Big[\phi+\lambda \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \mathbb{1}\big(S_{NF}(\boldsymbol{x},\boldsymbol{y})>0\big) + \sum_{\boldsymbol{x}'} \rho_u(x'|x)\Big] \\
&= \delta_{NF} \sum_{\boldsymbol{y},S>0} h_{NF}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{y},S<0} h_{NF}(\boldsymbol{x},\boldsymbol{y}) + \sum_{\boldsymbol{x}'} \rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{NF}(\boldsymbol{x}') + \gamma\Big[u_{NC}(\boldsymbol{x}) + u_{PL}(\boldsymbol{x}) + u_{YC}(\boldsymbol{x})\Big]
\end{aligned}
$$

$$
\begin{aligned}
h_{NC}(\boldsymbol{x},\boldsymbol{y}) \cdot \Bigg[\delta_{NC}+\chi+\gamma &+ \underbrace{s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \mathbb{1}\Big(S_{NC}(\boldsymbol{x},\boldsymbol{y}') > S_{NC}(\boldsymbol{x},\boldsymbol{y})\Big)}_{\text{leave for better jobs}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_e(x'|x,y)}_{\text{HC changes}}\Bigg] \\
&= \underbrace{\lambda\, v(\boldsymbol{y})\, u_{NC}(\boldsymbol{x})}_{\text{job acceptance}} + \underbrace{s\lambda\, v(\boldsymbol{y}) \sum_{\boldsymbol{y}'} h_{NC}(\boldsymbol{x},\boldsymbol{y}') \mathbb{1}\Big(S_{NC}(\boldsymbol{x},\boldsymbol{y}') < S_{NC}(\boldsymbol{x},\boldsymbol{y})\Big)}_{\text{poaching from other jobs}} + \underbrace{\sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{NC}(\boldsymbol{x}',\boldsymbol{y})}_{\text{HC changes}}
\end{aligned}
$$

$$
h_{PL}(\boldsymbol{x},\boldsymbol{y}) \cdot \Big[\delta_{YC}+\eta+\gamma\Big] = \chi\Big[h_{NC}(\boldsymbol{x},\boldsymbol{y}) + h_{YC}(\boldsymbol{x},\boldsymbol{y})\Big]
$$

$$
\begin{aligned}
h_{YC}(\boldsymbol{x},\boldsymbol{y}) \cdot \Big[\delta_{YC}+\chi+\gamma+s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \mathbb{1}\Big(S_{YC}(\boldsymbol{x},\boldsymbol{y}') > S_{YC}(\boldsymbol{x},\boldsymbol{y})\Big) + \sum_{\boldsymbol{x}'} \rho_e(x'|x,y)\Big] \\
&= \lambda\, v(\boldsymbol{y})\, u_{YC}(\boldsymbol{x}) + s\lambda\, v(\boldsymbol{y}) \sum_{\boldsymbol{y}'} h_{YC}(\boldsymbol{x},\boldsymbol{y}') \mathbb{1}\Big(S_{YC}(\boldsymbol{x},\boldsymbol{y}') < S_{YC}(\boldsymbol{x},\boldsymbol{y})\Big) + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{YC}(\boldsymbol{x}',\boldsymbol{y}) + \eta \cdot h_{PL}(\boldsymbol{x},\boldsymbol{y})
\end{aligned}
$$

$$
\begin{aligned}
h_{NF}(\boldsymbol{x},\boldsymbol{y}) \cdot \Big[\delta_{NF}+\phi+s\lambda \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \mathbb{1}\Big(S_{NF}(\boldsymbol{x},\boldsymbol{y}') > S_{NF}(\boldsymbol{x},\boldsymbol{y})\Big) + \sum_{\boldsymbol{x}'} \rho_e(x'|x,y)\Big] \\
&= \lambda\, v(\boldsymbol{y})\, u_{NF}(\boldsymbol{x}) + s\lambda\, v(\boldsymbol{y}) \sum_{\boldsymbol{y}'} h_{NF}(\boldsymbol{x},\boldsymbol{y}') \mathbb{1}\Big(S_{NF}(\boldsymbol{x},\boldsymbol{y}') < S_{NF}(\boldsymbol{x},\boldsymbol{y})\Big) + \sum_{\boldsymbol{x}'} \rho_e(\boldsymbol{x}|\boldsymbol{x}')\, h_{NF}(\boldsymbol{x}',\boldsymbol{y}) \\
&\quad + \gamma\Big[h_{NC}(\boldsymbol{x},\boldsymbol{y}) + h_{PL}(\boldsymbol{x},\boldsymbol{y}) + h_{YC}(\boldsymbol{x},\boldsymbol{y})\Big]
\end{aligned}
$$

Appendix G Smoothing shocks

Assume that the *iid* shock z follows a Logistic distribution with mean 0 and scale parameter ζ, so $\Gamma(z) = \frac{1}{1+e^{-z/\zeta}}$. Then the expected value from a job with surplus S has an analytical expression:

$$\begin{aligned}
&\int (S+z)\cdot \mathbb{1}[S+z>0]\, d\Gamma(z) \\
&= \zeta \, \log(1+e^{\frac{S}{\zeta}})
\end{aligned}$$

The expected value of unemployment in NC stage in equation (4) now becomes:

$$\begin{aligned}
\Big(r+\sum_{x'}\rho_u(\boldsymbol{x}'|\boldsymbol{x})+\chi+\gamma\Big)\, U^g_{NC}(\boldsymbol{x}) = b\,k + \sum_{x'}\rho_u(\boldsymbol{x}'|\boldsymbol{x})\, U^g_{NC}(\boldsymbol{x}') + \chi\, U^g_{PL}(\boldsymbol{x}) + \gamma\, U^g_{NF}(\boldsymbol{x}) \\
+ \lambda\,\beta \sum_{\boldsymbol{y}} v(\boldsymbol{y}) \int_{z>-S} \Big(S^g_{NC}(\boldsymbol{x},\boldsymbol{y})+z\Big) d\Gamma(z)
\end{aligned}$$

where $S^g_{NC}(\boldsymbol{x},\boldsymbol{y})$ denotes the expected surplus and does not depend on z.

Similarly, the expected values from job-to-job transitions are now slightly different in the joint values of a match, and equation (6) becomes:

$$\begin{aligned}
\Big(r+H(k,p)+\delta^g_{NC}+\chi+\gamma\Big)\, P^g_{NC}(\boldsymbol{x},\boldsymbol{y}) = (1-\tau)\, f(k,p) + q(\epsilon,\alpha) + \sum_{\boldsymbol{x}'}\rho_e(\boldsymbol{x}'|\boldsymbol{x},\boldsymbol{y})\, \widetilde{P}^g_{NC}(\boldsymbol{x}',\boldsymbol{y}) \\
+ \delta^g_{NC}\Big(\Pi_0(\boldsymbol{y}) + U^g_{NC}(\boldsymbol{x})\Big) + \chi\, \widetilde{P}^g_{PL}(\boldsymbol{x},\boldsymbol{y}) + \gamma\, \widetilde{P}^g_{NF}(\boldsymbol{x},\boldsymbol{y}) \\
+ s\lambda\,\beta \sum_{\boldsymbol{y}'} v(\boldsymbol{y}') \int_{z>S-S'} \Big(S^g_{NC}(\boldsymbol{x},\boldsymbol{y}') + z - S^g_{NC}(\boldsymbol{x},\boldsymbol{y})\Big) d\Gamma(z)
\end{aligned}$$

The balanced flow equations also take into account of the shocks. For example in NC stage, flows in and out of unemployment becomes:

$$\begin{aligned}
u_{NC}(\boldsymbol{x}) \cdot \Big[\chi+\gamma+\lambda\sum_{\boldsymbol{y}} v(\boldsymbol{y})\cdot \text{Prob}\big[S_{NC}(\boldsymbol{x},\boldsymbol{y})+z>0\big] + \sum_{\boldsymbol{x}'}\rho_u(x'|x)\Big] \\
= \delta_{NC}\sum_{\boldsymbol{y}} h_{NC}(\boldsymbol{x},\boldsymbol{y})\cdot \text{Prob}\big[S_{NC}(\boldsymbol{x},\boldsymbol{y})+z>0\big] + \sum_{\boldsymbol{y}} h_{NC}(\boldsymbol{x},\boldsymbol{y})\cdot \text{Prob}\big[S_{NC}(\boldsymbol{x},\boldsymbol{y})+z<0\big] \\
+ \sum_{\boldsymbol{x}'}\rho_u(\boldsymbol{x}|\boldsymbol{x}')\, u_{NC}(\boldsymbol{x}') + \xi_0(\boldsymbol{x})
\end{aligned}$$

where $\text{Prob}\big[S+z>0\big] = \frac{1}{1+e^{-S/\zeta}}$.

As the scaling parameter $\zeta \to 0$, the model converges to the discrete version. In practice, ζ is set to 0.5 in estimation and it is enough to ensure an equilibrium solution for a wide range of parameter values. Slightly changing the values of ζ does not alter the model moments by much, and thus does not affect parameter estimates.

Appendix H Estimation procedures and standard errors

I use the following iterative procedure to estimate two sets of parameters, the transition parameters $\mathbf{\Lambda} = (\delta^f_{NC}, \delta^f_{YC}, \delta, \vartheta, s_U, s_E)$ and the core parameters $\mathbf{\Theta} = (d_1, d_2, K, a, \sigma, \beta, b, \mu_m, \mu_f, M)$.

Step 1: Core moments given transition parameters Given a value for the transition parameters $\mathbf{\Lambda}$ obtained from the previous iteration (or an initial guess at the start), I estimate $\mathbf{\Theta}$ by minimizing the following quadratic distance

$$L_1(\mathbf{\Theta}|\mathbf{\Lambda}) = (\hat{m}^D_1 - \hat{m}^S_1(\mathbf{\Theta}|\mathbf{\Lambda}))^T \ \hat{W}_1^{-1} \ (\hat{m}^D_1 - \hat{m}^S_1(\mathbf{\Theta}|\mathbf{\Lambda}))$$

where $\hat{m}^D_1$ is a vector of data moments related to wage profiles of men and women, U-to-E wages and wage growths, proportion of men and women in high- and low-amenity jobs etc. that are described in section 4.3. The vector $\hat{m}^S_1$ are the corresponding model moments from simulations, taking $\mathbf{\Lambda}$ as given.

Step 2: Transition moments given core parameters Given the estimate of $\mathbf{\Theta}$ obtained from the previous step, I update the estimate of $\mathbf{\Lambda}$ by matching appropriate moments related to transitions:

$$L_2(\mathbf{\Lambda}|\mathbf{\Theta}) = (\hat{m}^D_2 - \hat{m}^S_2(\mathbf{\Lambda}|\mathbf{\Theta}))^T \ \hat{W}_2^{-1} \ (\hat{m}^D_2 - \hat{m}^S_2(\mathbf{\Lambda}|\mathbf{\Theta}))$$

I iterate over these two steps using MCMC until the functions L_1 and L_2 are minimized and the estimates of $\mathbf{\Lambda}$ and $\mathbf{\Theta}$ converge. The estimation strategy is a good fit for my problem because MCMC is derivative-free, so it is able to handle the non-linearities in the criterion functions due to the discreteness in the model. MCMC can also deal with large parameter spaces and multiple local minima quite well.[49]

I use the sandwich formula to estimate standard errors. Normally, the variance of the converged MCMC chain would provide a direct way to construct valid confidence intervals for the parameter estimates if the optimal weighting matrix is used. But I use a

[49]See the discussion in Chernozhukov and Hong (2003) for more details.

diagonally weighted approach. I will illustrate the computation for the core parameters $\mathbf{\Theta}$ below (the calculation is analogous for the transition parameters $\mathbf{\Lambda}$). The estimated covariance matrix has the form

$$\hat{V}(\hat{\mathbf{\Theta}}) = \Big(G'(\hat{\mathbf{\Theta}})\Omega G(\hat{\mathbf{\Theta}})\Big)^{-1} G'(\hat{\mathbf{\Theta}})\Omega\, \hat{E}\Big[(m_1^S(\hat{\mathbf{\Theta}}) - \hat{m}_1^D)(m_1^S(\hat{\mathbf{\Theta}}) - \hat{m}_1^D)'\Big]\Omega G(\hat{\mathbf{\Theta}})\, \Big(G'(\hat{\mathbf{\Theta}})\Omega G(\hat{\mathbf{\Theta}})\Big)^{-1}$$

where Ω is the weight matrix used in the estimation, $G(\hat{\mathbf{\Theta}})$ is the gradient matrix evaluated at the estimated parameters $\hat{\mathbf{\Theta}}$.

Estimates for the gradient G are obtained through simulation. Suppose m_1 consists of K moments and $\mathbf{\Theta}$ consists of J parameters. Then the numerical derivatives $\hat{G}(\hat{\mathbf{\Theta}})$ is a $K \times J$ matrix where the j-th column is computed as:

$$\hat{G}_j = \frac{m_1^S(\hat{\mathbf{\Theta}} + h\,\hat{\Theta}_j) - m_1^S(\hat{\mathbf{\Theta}} - h\,\hat{\Theta}_j)}{2\,h\,\hat{\Theta}_j}$$

where m_1^S is the vector of simulated moments evaluated at $\hat{\mathbf{\Theta}} + h\,\hat{\Theta}_j$ and $\hat{\mathbf{\Theta}} - h\,\hat{\Theta}_j$ respectively. The step size of deviation h is a vector of zeros except for one positive element at the j-th position equal to 1%. $\hat{\Theta}_j$ is the j-th element of $\hat{\mathbf{\Theta}}$.