Divorce with children: the partial efficiency approach

P.A. Chiappori[*], Monica Costa Dias[†], Costas Meghir[‡], Pengpeng Xiao[§]

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Abstract

We introduce a new concept called *partial efficiency* (PE) to model the post-divorce behaviors of ex-spouses. We assume that divorced parents still care about their children and maintain an efficient approach to the provision of the public good, but they do not share risk or compensate each other's private consumption. We show that the PE approach offers more realistic implications than the full-efficiency (FE) or non-cooperative (NC) models of divorce.

[*]Columbia University

[†]Bristol University, IFS

[‡]Yale University, IFS, NBER, IZA, CEPR

[§]Duke University

1 Introduction

The analysis of divorce and its welfare implications has recently attracted renewed attention from economists.[1] Not only is individual behavior after divorce (and its consequences in terms of poverty and inequality) interesting per se, but it may play a crucial role in understanding behavior when married, if only because it provides a possible threat point for any bargaining model of the married couple. Existing models have focused on how divorce outcomes affect behavior during marriage but have not focused on post-divorce interactions, imposing the simplifying assumption that following separation ex-spouses stop interacting with each other and go their separate ways (Voena, 2015; Fernández and Wong, 2017; Low, Meghir, Pistaferri *et al.*, 2022). Such an assumption, however, ignores a basic fact, namely that in most cases both divorced parents still care about their children and contribute to their welfare. In fact, a host of crucially important issues, such as the impact of divorce on child development, precisely depend on understanding how post-divorce interactions and decisions take place.

The goal of this note is to discuss the various modeling options available to explicitly analyze such a situation. We will focus on investments in child human capital, which we take to be a public good that the ex-spouses still enjoy following separation. We first argue that neither a standard collective model (Chiappori, 1988, 1992; Blundell, Chiappori, and Meghir, 2005) nor a non-cooperative approach provide an adequate framework for understanding these interactions. The collective model relies on an efficiency assumption, particularly in terms of risk sharing and private consumption, that contradicts the potentially adversarial nature of the relationship between ex-spouses. Non cooperative models, on the other hand, involve private provision of the public good, which has been known to deliver largely counterfactual predictions. We require a modelling framework that is capable of fitting the existing data patterns and to provide a way of understanding the underlying

[1] For some key contributions see Fernández and Wong (2017); Lafortune and Low (2017); Wolfers (2006); Stevenson (2007, 2008); Stevenson and Wolfers (2006); Voena (2015); Rasul (2005).

mechanisms that drive investments and ultimately outcomes for children of divorced couples. This is particularly important given the need to understand why children of divorced parents appear to have worse outcomes, which could be a combination of selection effects and changes in parental investments.

We introduce a new concept called *partial efficiency*. In a partially efficient context, individuals maintain an efficient approach to the provision of the public good; in particular, their contribution to public expenditures takes into account the benefits derived by the ex-spouse. Regarding all other economic decisions, however, individuals fail to cooperate: they do not share risk or compensate each other's private consumptions. We show that the partial efficiency approach differs from both the fully cooperative and the non cooperative framework, with distinct empirical implications. For example, contrary to the concept of partial efficiency that we will now introduce, the non-cooperative model implies income pooling when both ex-partners are contributing to the public good, while the cooperative model implies full insurance between them even post divorce. Both these implications can be rejected with post-divorce data on private and public expenditures.

2 The model

2.1 Basic version: no domestic production

We consider a couple consuming four commodities: two individual leisures L^a, L^b a public good Q (that could be interpreted, depending on the context, as expenditures on children, children's human capital or more globally as children's welfare), and a Hicksian private good c for which individual consumptions are not observed. Individual preferences are summarized by a utility of the form $U^i\left(c^i, L^i, Q\right)$, $i = a, b$. When married, we assume individuals reach Pareto-efficient agreements. Their behavior is thus represented by a standard, collective setting:

$$\max U^a\left(c^a, L^a, Q\right) + \mu U^b\left(c^b, L^b, Q\right) \tag{1}$$

under the budget constraint

$$c^a + c^b + PQ + w^a L^a + w^b L^b = \left(w^a + w^b\right) T + y$$

Here, w^i denotes i's wage, T is total time available, y is the couple's non labor income and μ is a Pareto weight.

We now assume that the individuals under consideration have divorced; that is, U^i now denotes post-divorce individual utilities, which may or may not be identical to pre-divorce ones. We can assume that non labor income y is then divided between the spouses - i.e., individual i receives y^i, with $y^a + y^b = y$. Note that y^i may be negative (then $y^j, j \neq i$, exceeds y); that would be the case, for instance, if the divorce settlement involves part of i's labor income be transferred to j. Crucially, commodity Q remains public after divorce; that is, both ex-spouses still care about children's welfare, although *how much* they do may be affected by divorce.

In order to model the ex-spouses' decision process, we successively consider three possible settings.

2.1.1 Full efficiency

A first approach would assume that individuals still reach a fully efficient (from now on FE) agreement. Then they jointly solve a problem of the type (1); the only difference with the pre-divorce situation is that individual utilities and the Pareto weight may have changed after divorce. Note, in particular, that they face a unique, common budget constraint. In practice, thus, the ex-spouses jointly solve the program

$$\max U^a \left(c^a, L^a, Q^a + Q^b\right) + \mu U^b \left(c^b, L^b, Q^a + Q^b\right) \tag{2}$$

under

$$c^a + w^a L^a + c^b + w^b L^b + PQ = Y^a + Y^b$$

where Y^i denotes i's potential income:

$$Y^i = w^i T + y^i, \quad i = a, b$$

Equivalently, an efficient decision can always be represented as stemming from a two-stage process. In stage 1, individuals jointly choose the quantity of the public good and a *sharing rule* ρ that defines how the remaining income is split between them. In stage 2, they independently decide on their labor supply and private consumption, under the budget constraint defined by the sharing rule. In practice, the second stage decision of individual i solves:

$$V^i \left(w, \rho^i, Q\right) = \max_{c,L} U^i \left(c, L, Q\right) \text{ under } c + wL = \rho^i$$

The function V^i is called the conditional indirect utility of i.[2] Finally, the first stage program is:

$$\max_{Q,\rho^a,\rho^b} V^a \left(w^a, \rho^a, Q\right) + \mu V^b \left(w^b, \rho^b, Q\right) \text{ under } \rho^a + \rho^b + PQ = Y = Y^a + Y^b,$$

and the first order conditions give the usual, Bowen-Lindahl-Samuelson equations:

$$MWP^a + MWP^b = \frac{\partial V^a / \partial Q}{\partial V^a / \partial \rho^a} + \frac{\partial V^b / \partial Q}{\partial V^b / \partial \rho^b} = P$$

expressing the fact that individual marginal willignesses to pay for the public good add up to its price.

[2] In the particular case of separable preferences:

$$U^i \left(c, L, Q\right) = \bar{U}^i \left(u^i \left(c, L\right), Q\right)$$

the first stage only affects individual decisions in stage 2 through the budget constraint. In general, however, the MRS between leisure and private consumption are also affected by public expenditures; in that sense, the indirect utility is defined conditionally on Q.

This solution, however, requires a level of cooperation that may not be realistic for divorced individuals. Assume, in particular, that the solution to the previous program, $\left(\bar{c}^a, \bar{L}^a, \bar{c}^b, \bar{L}^b, \bar{Q}\right)$, is such that

$$\bar{c}^i + w^i \bar{L}^i > w^i T + y^i - P\bar{Q} \text{ for some } i.$$

Then the program involves transfers from j to i that exceed those mandated by the divorce settlement (the later being summarized by the y^is); the full efficiency model implicitly assumes that such transfers are fully implementable.

2.1.2 Non cooperation (NC)

Alternatively, we may assume that ex-spouses choose not to cooperate. Since public good expenditures still enter both utlities, implying that they both may want to contribute to it, the corresponding game is a private contribution one in which ex-spouses each choose their contribution taking the other spouse's as given.

The properties of the corresponding Nash equilibria are by now well-known (Bergstrom, Blume, and Varian, 1986; Browning, Chiappori, and Lechene, 2009). It can take either of two forms. Either it corresponds to a corner solution, whereby the public good is entirely funded by one spouse only. In practice, that would imply that one ex-spouse (the custodial parent in general) takes in charge the full amount of children expenditures, with no help whatsoever from the other. While such situations are by no means unheard of, they can hardly be considered as the standard outcome of divorce, let alone the only possible one.

Alternatively, the solution may be interior, in the sense that both individuals contribute to the funding. In that case, however, the resulting equilibrium exhibits a strong *income pooling* property. Namely, the allocation, including the distribution of private consumption among ex-spouses, only depends on total (ex-) household resources, not on its distribution between ex-spouses. In particular, a change in the divorce settlement - say, an increase in y^a

and a corresponding decrease in y^b - cannot possibly have any impact on individual behavior or well-being. Again, such a conclusion appears to be largely counterfactual. We conclude that the non-cooperative approach to post-divorce behavior is essentially inadequate.

2.1.3 Partial efficiency: definition

We now introduce a third setting which avoids the problems raised by the previous two. Just like FE, partial efficiency (from now on PE) can be seen as the outcome of a two-stage process. In stage one, ex-spouses jointly decide on the amount to be spent on the public good and on each ex-spouse's contribution; the latter can be either monetary, in time, or both. Importantly, the decision is assumed to be efficient, in the (usual) sense that it maximizes a weighted sum of individual utilities under a budget constraint. In stage two, individuals each choose their consumption and market labor supply under the constraints defined by stage one.

A crucial difference, however, is that in a FE context stage 1 determines both the quantity of the public good *and* the sharing rule; the latter will in turn determine individual (market) labor supplies and private consumptions. Under PE, stage 1 does not fix a sharing rule, but simply individual contributions to the public good - the latter being moreover subject to a non negativity constraint.

In others words, in a PE context decisions on public good expenditures explicitly consider the benefits derived by *both* ex-partners - this is the efficiency part. Efficiency, however, is only partial, because *direct* transfers across couples are ruled out (beyond those implied by the divorce settlement); as a consequence, ex-spouses each face their own budget constraint, which may or may not affect efficiency.[3]

The formal translation of these ideas depends on the context, and in particular on the presence of domestic production. We first consider the case of purely monetary contributions;

[3] For instance, risk sharing mechanisms are strongly limited by this assumption.

domestic production will be analyzed in the next subsection. The model is solved backwards. In stage 2, let $\left(Q^a, Q^b\right)$ denotes individual contributions to the public good, as defined in the previous stage, and let $Q = Q^a + Q^b$. Individual i then solves the following program:

$$V^i\left(w, Y^i - PQ^i, Q\right) = \max_{c,L} U^i\left(c, L, Q\right) \text{ under } c + wL = \rho^i$$

under the budget constraint

$$c^i + w^i L^i = Y^i - PQ^i$$

and the time allocation constraint $L^i \leq T$, where T denotes total available time and V^i is again the conditional indirect utility.

Next, the collective decision process at stage 1 is summarized by the following program:

$$\max_{Q^a, Q^b} V^a\left(w^a, Y^a - PQ^a, Q^a + Q^b\right) + \mu V^b\left(w^b, Y^b - PQ^b, Q^a + Q^b\right) \tag{3}$$

under the constraints $Q^i \geq 0, i = a, b$.

We see, in particular, that while direct monetary transfers between ex-spouses are ruled out, the allocation of public good expenditures across ex-spouses is unconstrained. In particular, *implicit* transfers via changes in the individuals' respective contributions are possible. The only constraint is non negativity; in practice, it requires that an individual's *private* consumption (including leisure) cannot exceed the individual's potential income.

Two cases must therefore be considered. If the solution to the full efficiency framework, $\left(\bar{c}^a, \bar{L}^a, \bar{c}^b, \bar{L}^b, \bar{Q} = \bar{Q}^a + \bar{Q}^b\right)$, is such that

$$\bar{c}^i + w^i \bar{L}^i \leq Y^i \text{ for all } i, \tag{4}$$

then one can define i's contribution $\bar{Q}^i$ by

$$P\bar{Q}^i = Y^i - \left(\bar{c}^i + w^i \bar{L}^i\right)$$

It follows that the full efficiency solution is compatible with the partial efficiency constraints, and partial efficiency boils down to full efficiency.

In the alternative case, one of the constraints - say, for a - is violated. Then a corner solution obtains. That is, a's second stage program becomes:

$$V^a\left(w^a, Y^a, Q\right) = \max_{c^a, L^a} U^a\left(c^a, L^a, Q\right) \text{ under } c^a + w^a L^a = Y^a, \tag{5}$$

while the program of b is

$$V^b\left(w^b, Y^b - PQ, Q\right) = \max_{c^b, L^b} U^b\left(c^b, L^b, Q\right) \text{ under } c^b + w^b L^b = Y^b - PQ \tag{6}$$

The first stage program becomes:

$$\max_Q V^a\left(w^a, Y^a, Q\right) + \mu V^b\left(w^b, Y^b - PQ, Q\right) \tag{7}$$

The first order conditions are now:

$$\frac{\partial V^a}{\partial Q} + \mu \frac{\partial V^b}{\partial Q} = \mu P \frac{\partial V^b}{\partial y} \text{ or} \tag{8}$$

$$\frac{1}{\mu} \frac{\partial V^a / \partial Q}{\partial V^b / \partial y} + \frac{\partial V^b / \partial Q}{\partial V^b / \partial y} = P \tag{9}$$

which fails to be efficient since

$$\frac{\partial V^a}{\partial y} \neq \mu \frac{\partial V^b}{\partial y}$$

at a corner solution.

The interpretation is straightforward. The constraint (4) is now binding for $i = a$, implying that the ratio of marginal utilities of income differs from the Pareto weight. In practice, the solution is now inefficient, since a's private consumptions are less than what the full efficiency model would imply.

Two points may be noted here:

- The partial efficiency solution typically generates more public goods expenditures than the non cooperative one. This simply reflects the fact that, when choosing this level, b also takes into account the utility a derives from public good expenditures.
- More surprinsingly, the partial efficiency solution typically generates more public goods expenditures than the full efficiency one - at least when they differ. This intuition, here, is that the two concepts differ when one individual constraint is binding - i.e., when the FE solution would imply more private consumption for one individual than what the individual's personnal budget constraint allows. With well-behaved utilities, this results in less private expenditures for that individual, therefore more public expenditures overall, under PE than under FE. Note, however, that this conclusion heavily depends on the absence of domestic production, as can be seen below.

2.2 Domestic production

We now consider a different model in which the public good is produced within the household - a situation the fits the interpretation in terms of children welfare. Specifically, we now assume that

$$Q = \phi\left(t^a, t^b\right)$$

where t^i denotes i's time devoted to domestic production[4]. In most cases, the output Q is not directly observed, implying that the scale of the process is unobservable; only the inputs

[4] A more general case would involve both time and money as inputs to the production function. Then solutions can either interior or corner.

t^a and t^b are recorded (say, from time use data). Then a natural assumption is that the technology described by ϕ exhibits constant returns to scale:

$$\phi\left(kt^a, kt^b\right) = k\phi\left(t^a, t^b\right) \text{ for all } k > 0$$

implying that

$$\phi\left(t^a, t^b\right) = t^b \psi\left(\frac{t^a}{t^b}\right) \text{ for some } \psi$$

As a consequence the partial derivatives of ϕ only depend on the ratio t^a/t^b:

$$\frac{\partial \phi}{\partial t^a} = \psi'\left(\frac{t^a}{t^b}\right), \frac{\partial \phi}{\partial t^b} = \psi - \frac{t^a}{t^b}\psi'\left(\frac{t^a}{t^b}\right)$$

2.2.1 Full efficiency

As before, we start with the full efficiency benchmark. Here, the ex-spouses jointly solve

$$\max U^a\left(c^a, L^a, \phi\left(t^a, t^b\right)\right) + \mu U^b\left(c^b, L^b, \phi\left(t^a, t^b\right)\right) \tag{10}$$

under

$$c^a + w^a L^a + c^b + w^b L^b = Y^a + Y^b$$

First order conditions are:

$$\frac{\partial U^a}{\partial c^a} = \lambda, \mu\frac{\partial U^b}{\partial c^b} = \lambda, \frac{\partial U^a}{\partial L^a} = \lambda w^a, \mu\frac{\partial U^b}{\partial L^b} = \lambda w^b$$

and

$$\left(\frac{\partial U^a}{\partial Q} + \mu\frac{\partial U^b}{\partial Q}\right)\frac{\partial \phi}{\partial t^a} = \lambda w^a, \left(\frac{\partial U^a}{\partial Q} + \mu\frac{\partial U^b}{\partial Q}\right)\frac{\partial \phi}{\partial t^b} = \lambda w^b \tag{11}$$

where λ denotes the Lagrange multiplier of the budget constraint. Thus FE implies a specific version of the Bowen-Lindahl-Samuelson conditions:

$$\left(MWP^a + MWP^b\right) \frac{\partial \phi}{\partial t^x} = w^x, \quad x = a, b$$

In particular, we have that:

$$\frac{\partial \phi / \partial t^b}{\partial \phi / \partial t^a} = \frac{w^b}{w^a} \tag{12}$$

This equation pins down the ratio t^a/t^b. In particular, *the latter does not depend on* μ. The interpretation is clear. The *scale* of public good production (represented in that setting by the level of public good production Q) depends on the intra-household power allocation: more power to the spouse who 'cares more' about the public good (in the sense defined by Blundell, Chiappori, and Meghir, 2005) results in more production of it. For a given scale, however, the allocation of inputs is totally driven by productive efficiency; it is simply the cheapest way to produce the desired level. In particular, the ratio of a and b's time contributions is not affected by the intra-household balance of power; under the CRS assumption, it is fully determined by the ratio of individual wages (and the production technology).[5]

2.2.2 Non cooperation

We next consider the non-cooperative framework, with individuals each choosing their domestic and market labor supply while taking the ex-spouse's contribution to domestic production as given. First order conditions for individual x ($x = a, b$) are

$$\frac{\partial U^x}{\partial c^x} = \lambda^x, \quad \frac{\partial U^x}{\partial L^x} = \frac{\partial U^x}{\partial Q} \frac{\partial \phi}{\partial t^x} = \lambda^x w^x \tag{13}$$

where λ^x is x's marginal utility of income. Two conclusions emerge:

[5] This point suggests, in particular, that the spouses' respective contributions to domestic production need not reflect the distribution of power within the household; it all depends on the type of efficiency assumed to be achieved by the decision process.

- The public good is underproduced; indeed, (13) implies that

$$MWP^x \frac{\partial \phi}{\partial t^x} = w^x, \quad x = a, b \tag{14}$$

Intuitively, individuals each fail to consider the benefit their investment will provide to the spouse.

- Moreover, this (insufficient) amount of public good is *inefficiently* produced, in the sense that the same level of public good could be produced at a lower total cost (thus potentially allowing more private leisure and consumption for both spouses). Indeed, (14) implies that

$$\frac{\partial \phi / \partial t^b}{\partial \phi / \partial t^a} = \frac{w^b}{w^a} \times \frac{MWP^a}{MWP^b} \neq \frac{w^b}{w^a}$$

In words, if $MWP^a > MWP^b$, then a invests too much (and b too little) time into public good production, as compared to what productive efficiency would require.

2.2.3 Partial efficiency

Finally, partial efficiency still relies on a two stage process. In stage 2, individual i solves

$$V^i \left(w^i, Y^i - w^i t^i, \phi\left(t^a, t^b\right)\right) = \max_{c^i, L^i} U^i \left(c^i, L^i, \phi\left(t^a, t^b\right)\right), \tag{15}$$

under $L^i \leq 1 - t^i$ and

$$c^i + w^i L^i = Y^i - w^i t^i$$

where V^i is again i's conditional indirect utility. The first stage is therefore

$$\max_{t^a, t^b} V^a \left(w^a, Y^a - w^a t^a, \phi\left(t^a, t^b\right)\right) \tag{16}$$

$$+ \mu V^b \left(w^b, Y^b - w^b t^b, \phi\left(t^a, t^b\right)\right) \tag{17}$$

under $0 \leq t^i \leq T$, which gives the following first order conditions:

$$\begin{aligned}\left(\frac{\partial V^a}{\partial Q} + \mu \frac{\partial V^b}{\partial Q}\right) \frac{\partial \phi}{\partial t^a} &= w^a \frac{\partial V^a}{\partial y} \text{ and} \\ \left(\frac{\partial V^a}{\partial Q} + \mu \frac{\partial V^b}{\partial Q}\right) \frac{\partial \phi}{\partial t^b} &= w^b \frac{\partial V^b}{\partial y}\end{aligned}$$

In particular,

$$\frac{\partial \phi / \partial t^b}{\partial \phi / \partial t^a} = \frac{\partial V^b / \partial y}{\partial V^a / \partial y} \times \frac{w^b}{w^a} \neq \frac{w^b}{w^a}$$

Again, the respective time contributions depend not only on wages, but also on the Pareto weight and the post-divorce allocation of income (through the corresponding marginal utilities of income). In particular, *productive efficiency does not obtain:* for almost all outcomes, it would have been possible to achieve the same level of public good at a lower (total) cost - but that would have required transfers between spouses that are not feasible under PE. The production level of public goods, however, could be either lower or higher than the efficient level, as illustrated by the following example.

3 A Cobb-Douglas example

We now illustrate the previous results by considering specific preferences. Namely, assume that utilities are Cobb-Douglas:

$$U^i\left(c^i, L^i, Q\right) = \alpha^i \ln L^i + \beta^i \ln Q + \left(1 - \alpha^i - \beta^i\right) \ln c^i$$

so that the weighted sum of utilities is also CD:

$$U^a + \mu U^b = \alpha^a \ln L^a + (1 - \alpha^a - \beta^a) \ln c^a + \mu \alpha^b \ln L^b + \mu\left(1 - \alpha^b - \beta^b\right) \ln c^b + \left(\beta^a + \mu \beta^b\right) \ln Q$$

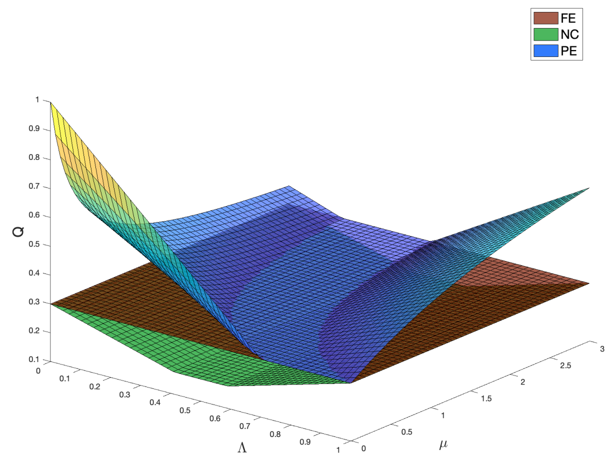

Figure 1: Expenditure on the public good as a function as income share and the Pareto weight

In what follows, we let Y denote total household income, and Λ the fraction coming from A:

$$\Lambda = Y^a/Y$$

3.1 Basic version: no domestic production

We start with the basic model.[6] The global budget constraint is

$$c^a + c^b + w^a L^a + w^b L^b + PQ = Y$$

Figure 1 gives the level of public expenditures (as a fraction of total income Y) for FE, NC and PE, as functions of the Pareto weight μ and of the income ratio Λ. We see that, as expected, non cooperation reduces public good expenditures vis a vis both full and partial efficiency. The latter two cases may coincide; this happens when total optimal private expenditure for each individual is such that no transfers are required between the ex-spouses,

[6]Precise derivations can be found in Appendix.

i.e.:

$$c^a + w^a L^a = \frac{1-\beta^a}{1+\mu} Y \leq Y^a \text{ and } c^b + w^b L^b = \mu \frac{1-\beta^b}{1+\mu} Y \leq Y^b$$

However, if transfers are required to implement the first best under FE (i.e., when either $\frac{1-\beta^a}{1+\mu} Y > Y^a$ or $\mu \frac{1-\beta^b}{1+\mu} Y > Y^b$), partial efficiency (where such transfers are not feasible) results in more public good expenditures than full efficiency, as discussed above. This counterintuitive result is dependent on public goods being simply purchased in the market. Once we introduce domestic production this may no longer be true.

3.2 Domestic production

We now consider the case of domestic production; to keep things simple, we assume that the production function is also Cobb-Douglas:

$$\ln Q = \frac{1}{2} \left(\ln t^a + \ln t^b \right)$$

which satisfies the Constant Return to Scale assumption. The global budget constraint is now:

$$c^a + c^b + w^a \left(L^a + t^a \right) + w^b \left(L^b + t^b \right) = Y^a + Y^b = Y \tag{18}$$

Here, $Y^i = w^i T + y^i$ where T denotes total time available and y^i denotes i's non labor income (including possibly divorce settlements).

While the derivations are left for the Appendix, we state the main results. With full efficiency the time inputs only depend on the wage ratio, namely

$$\frac{t^a}{t^b} = \frac{w^b}{w^a}$$

However, under partial efficiency the ratio of time inputs deviates from productive efficiency and depends on the Pareto weight and preferences, as well as the wage ratio:

$$\frac{t^a}{t^b} = \frac{w^b}{w^a} \frac{2\mu - \left(\mu\beta^b - \beta^a\right)}{2 + \left(\mu\beta^b - \beta^a\right)} \frac{Y^a}{Y^b}$$

Finally, non cooperative behavior leads to a ratio of time inputs

$$\frac{t^a}{t^b} = \frac{w^b}{w^a} \frac{\beta^a \left(2 - \beta^b\right)}{\beta^b \left(2 - \beta^a\right)} \frac{Y^a}{Y^b}$$

To summarize: under full efficiency the overall inputs to the public good may, after separation, either fall or increase, depending on the resulting changes in overall resources, respective bargaining powers or individual preferences for the public good. However, the latter will be produced efficiently; as a result, both domestic time will either simultaneously increase or simultaneously decrease. Under constant returns, the ratio of time inputs of the two partners will not change, as it is determined by (and, in the Cobb-Douglas case, equal to) the ratio of their market wage.

Once we depart from full efficiency, and move either to a non-cooperative equilibrium or to our new concept of partial efficiency, the inputs get distorted away from efficient provision. This can for instance lead to one of the partners providing more than the efficient benchmark while the other provides less; this outcome depends on their new configuration of individual incomes, preferences and (under PE) relative bargaining power. The overall provision of the public good will be different from both the pre- and post-divorce efficient outcomes; but whether it is higher or lower will depend on the parameter configuration.

3.3 The comparative statics of divorce

From a theory perspective, divorce may have various types of impacts. Under (full or partial) efficiency, divorce may change the spouses' respective bargaining powers, as translated into changes in the Pareto weights. Moreover, the household may switch from fully to partially efficient agreements - this is indeed one of the main motivations for the partial efficiency concept. Last but not least, divorce may affect individual preferences toward children, particularly for the non custodial parents (who may 'benefit less' from the public good).

We now investigate the empirical implications of such changes. For brevity, we concentrate on the domestic production model, and we stick to Cobb-Douglas preferences, although most conclusion are valid more generally.

3.3.1 Changes in Pareto weights

We start with changes in Pareto weights in either a full or a partial efficiency context. Keeping a's weight normalized to 1, consider an increase in b's weight μ. Under both FE and PE, b's leisure and private consumption increase, while a's decrease. Regarding market work, it decreases for b and increases for a under PE. The FE case is more complex; yet, if one assumes that preferences for public good are 'not too different' - technically, if

$$\left|\beta^a - \beta^b\right| < \min\left(\alpha^a, \alpha^b\right),$$

then the same conclusion obtains.

The main difference is related to domestic time. Under FE, changes are totally driven by preferences for the public good. If b 'cares more' about the public good than a does[7] - that is, in the Cobb-Douglas case, if $\beta^b > \beta^a$ - then more power to b tends to increase the total production of public good. As a result, *both domestic times increase* (while their ratio remain constant). If $\beta^b < \beta^a$, the opposite conclusion obtains. Importantly, t^a and t^b always

[7] Again in the sense of Blundell, Chiappori, and Meghir (2005).

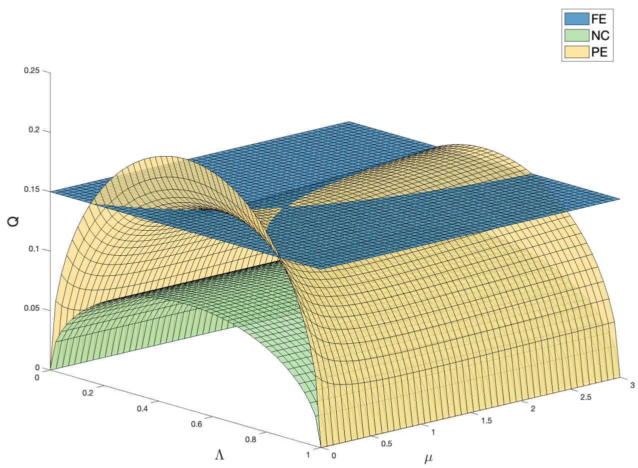

Figure 2: Public good provision with domestic production under FE, PE and NC

change *in the same direction*.

The PE case is totally different: when μ gets larger, a's domestic time always increases while b's always decreases, and the impact on total domestic production is ambiguous. Note that, in sharp contrast with the FE case, when Pareto weights change under PE then individual domestic times always move in *opposite* directions.

3.3.2 Changing regime: from Full to Partial Efficiency

As mentioned above, under PE productive efficiency is almost never achieved: one domestic labor supply almost always exceeds the efficient amount while the other is then below the efficient level.

As an example, Figure 2 gives, under a specific parameterization of preferences, the level of public good production for the three regimes (FE, PE and non cooperative); they are plotted as functions of the Pareto weight μ and the fraction Λ of the post-divorce income going to ex-spouse a. Partial efficiency almost never coincides with full efficiency; but public good production under PE may either exceed or fall short of the optimal level. When post-divorce total incomes are similar and μ is unbalanced (either very small of very large), the efficient allocation would require one of the ex-spouses (the one favored by a higher Pareto weight) to reach a level of private consumption that is significantly larger than what their budget constraint allows under PE. As a result, and just as in the no production case, the private consumption of that individual is suboptimal, leading to higher public consumption. In most cases, however, productive inefficiencies kick in and dominate, resulting in *suboptimal* levels of production. Finally, the public good production is always (largely) suboptimal under non cooperation, due to suboptimal investment by both parents.

3.3.3 Changes in preferences

Finally, what are the consequences of a decrease in a spouse's (say, a's) preferences for the public good? For example, in the case where the human capital of the child is the public good, the distance between the non-custodial parent and the child may reduce their attachment, which we capture here by a reduction in the utility weight of the public good in the non-custodial's utility function. A technical issue is how the decrease affects the MRS between consumption and leisure. In what follows, we simply consider a decrease in β^a that keeps α^a constant (thus increasing preferences for consumption); the main qualitative conclusions are unaffected. The main implications are as follows.[8]

- Under FE, both domestic times decrease, and their ratio remain equal to the wage ratio. The intuition is clear. Decreasing β^a while keeping β^b constant reduces the total weight of the public good in the household's maximization program, resulting in a lower production level; productive efficiency then requires that individual changes be inversely proportional to wages

- PE is more complex. Again, the total weight of the public good decreases, which leads to a reduction in both domestic times. However, the ratio of domestic times is also affected (specifically, t^a/t^b increases); moreover, the impact also depends on post-divorce allocations

[8]Precise derivations can be found in Appendix.

- Finally, under non cooperation, a's domestic time is reduced; in the Cobb-Douglas case, b's is unaffected.

3.4 Concluding Remarks: The consequences of divorce - what do we expect?

For understanding the effects of divorce on children and even more so the mechanisms that underlie these effects, we need a modelling framework than can fit the complex changes in behavior when a couple separates. In this short note we have outlined alternative approaches to modelling behavior and we have argued that the most flexible and intuitively reasonable approach is that of partial efficiency. Neither the fully efficient outcome nor the non-cooperative one can capture the richness of responses we observe in the data.

Divorce is a complex phenomenon, which may affect the couple's behavior in several ways. It may change individual respective incomes and bargaining powers, which in turn influence the spouses' allocation of time between leisure, domestic and market work. It might also modify preferences, for instance by decreasing the value put on public consumption by one of the ex-partners (typically the non custodial parent). Finally, it is likely to change the nature of the intra-household decision process away from efficiency. Interestingly, the various alternative assumptions about post-divorce behavior lead to different empirical predictions on responses. Specifically:

- The FE model has very clear-cut implications. Whether the changes affect preferences or Pareto weights, if full efficiency prevails both before and after divorce, then both domestic times should always move in the same direction; under constant returns, their ratio should moreover remain constant (and determined by the wage ratio). In particular, the gender asymmetry in behavior often observed after divorce (whereby, in many households, the father's domestic time decreases while the mother's increases or remains constant) seems incompatible with this model.

- If, alternatively, divorce leads to a switch from FE to NC, one should expect a drastic decrease in both domestic labor supplies. Again, this prediction does not seem consistent with observed empirical patterns. In addition, when both individuals contribute financially to the public good, their choices should be invariant to the distribution of income, another counterfactual prediction.

- The most promising approach, therefore, involves a shift from FE to PE, associated with a decrease in one parent's preferences for the public good. The regime change always increases one domestic time while decreasing the other, the impact on both total time and production level being ambiguous. If, in addition, preferences change as well, the decrease in one ex-partner's time will be exacerbated, whereas the increase in the ex-spouse's time will be mitigated (but will not necessarily disappear).

References

Bergstrom, T., Blume, L., and Varian, H. (1986). 'On the private provision of public goods.' *Journal of Public Economics*, vol. 29(1):pages 25–49. ISSN 0047-2727.

Blundell, R., Chiappori, P., and Meghir, C. (2005). 'Collective Labor Supply with Children.' *Journal of Political Economy*, vol. 113(6):pages 1277–1306. ISSN 0022-3808. Publisher: The University of Chicago Press.

Browning, M., Chiappori, P., and Lechene, V. (2009). 'Distributional Effects in Household Models: Separate Spheres and Income Pooling.' *The Economic Journal*, vol. 120(545):pages 786–799. ISSN 0013-0133.

Chiappori, P.-A. (1988). 'Rational household labor supply.' *Econometrica: Journal of the Econometric Society*, pages 63–90.

Chiappori, P.-A. (1992). 'Collective labor supply and welfare.' *Journal of Political Economy*, pages 437–467.

Fernández, R. and Wong, J. C. (2017). 'Free to Leave? A Welfare Analysis of Divorce Regimes.' *American Economic Journal: Macroeconomics*, vol. 9(3):pages 72–115.

Lafortune, J. and Low, C. (2017). 'Tying the Double-Knot: The Role of Assets in Marriage Commitment.' *American Economic Review*, vol. 107(5):pages 163–67.

Low, H., Meghir, C., Pistaferri, L., and Voena, A. (2022). 'Marriage, labor supply and the dynamics of the social safety net.' *NBER, WP w24356*.

Rasul, I. (2005). 'Marriage Markets and Divorce Laws.' *The Journal of Law, Economics, and Organization*, vol. 22(1):pages 30–69. ISSN 8756-6222.

Stevenson, B. (2007). 'The Impact of Divorce Laws on Marriage-Specific Capital.' *Journal of Labor Economics*, vol. 25(1):pages 75–94.

Stevenson, B. (2008). 'Divorce Law and Women's Labor Supply.' *Journal of Empirical Legal Studies*, vol. 5(4):pages 853–873.

Stevenson, B. and Wolfers, J. (2006). 'Bargaining in the Shadow of the Law: Divorce Laws and Family Distress*.' *The Quarterly Journal of Economics*, vol. 121(1):pages 267–288. ISSN 0033-5533.

Voena, A. (2015). 'Yours, Mine and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?' *American Economic Review*.

Wolfers, J. (2006). 'Did Unilateral Divorce Laws Raise Divorce Rates? A Reconciliation and New Results.' *American Economic Review*, vol. 96(5):pages 1802–1820.

4 Appendix: Cobb-Douglas preferences

4.1 Basic version: no domestic production

We start with the basic model. The global budget constraint is

$$c^a + c^b + w^a L^a + w^b L^b + PQ = Y^a + Y^b = Y$$

In what follows, we let Λ denote the ratio of a's income to total income:

$$\Lambda = Y^a/Y$$

4.1.1 Full efficiency

In the full efficiency benchmark, the program is

$$\max \alpha^a \ln L^a + (1-\alpha^a-\beta^a)\ln c^a + \mu\alpha^b \ln L^b + \mu\left(1-\alpha^b-\beta^b\right)\ln c^b + \left(\beta^a+\mu\beta^b\right)\ln Q$$

giving the solution:

$$c^a = \frac{1-\alpha^a-\beta^a}{1+\mu}Y, c^b = \frac{\mu\left(1-\alpha^b-\beta^b\right)}{1+\mu}Y, w^a L^a = \frac{\alpha^a}{1+\mu}Y, \quad w^b L^b = \frac{\mu\alpha^b}{1+\mu}Y$$
$$\text{and } PQ = \frac{\beta^a+\mu\beta^b}{1+\mu}Y$$

and individual utilities:

$$U^a = K^a + \ln Y - \alpha^a \ln w^a - \beta^a \ln P,$$
$$U^b = K^b + \ln Y - \alpha^b \ln w^b - \beta^b \ln P$$

where

$$K^a = \alpha^a \ln\left(\frac{\alpha^a}{1+\mu}\right) + \beta^a \ln\left(\frac{\beta^a+\mu\beta^b}{1+\mu}\right) + (1-\alpha^a-\beta^a)\ln\left(\frac{1-\alpha^a-\beta^a}{1+\mu}\right) \text{ and}$$
$$K^b = \alpha^b \ln\left(\frac{\mu\alpha^b}{1+\mu}\right) + \beta^b \ln\left(\frac{\beta^a+\mu\beta^b}{1+\mu}\right) + \left(1-\alpha^b-\beta^b\right)\ln\left(\frac{\mu\left(1-\alpha^b-\beta^b\right)}{1+\mu}\right)$$

4.1.2 Non cooperative

Next, consider the non cooperative outcome. For an interior solution, we have:

$$PQ^a = \frac{\beta^a Y^a - (1-\beta^a)\beta^b Y^b}{1-(1-\beta^a)(1-\beta^b)}, PQ^b = \frac{\beta^b Y^b - \left(1-\beta^b\right)\beta^a Y^a}{1-(1-\beta^a)(1-\beta^b)}$$

therefore

$$PQ = \frac{\beta^a \beta^b Y}{1-(1-\beta^a)(1-\beta^b)}$$

which, as expected, satisfies income pooling, since the solution only depends on total income Y, not on individual incomes Y^a and Y^b.

Corner solutions, on the other hand, obtain when the previous solution is such that either

$$PQ^a = Y^a\beta^a - Y^b\beta^b(1-\beta^a) \leq 0 \Rightarrow \Lambda \leq \frac{\beta^b(1-\beta^a)}{\beta^a + \beta^b(1-\beta^a)}$$

$$PQ^b = Y^b\beta^b - Y^a\beta^a\left(1-\beta^b\right) \leq 0 \Rightarrow \Lambda \geq \frac{\beta^b}{\beta^a(1-\beta^b)+\beta^b}$$

In the first case, $Q^a = 0$, then

$$w^a L^a = \frac{\alpha^a}{1-\beta^a} Y^a, c^a = \frac{1-\alpha^a-\beta^a}{1-\beta^a} Y^a$$

and

$$w^b L^b = \alpha^b Y^b, c^b = \left(1-\alpha^b-\beta^b\right) Y^b \text{ and } \frac{PQ}{Y} = \frac{PQ^b}{Y} = (1-\Lambda)\beta^b - \Lambda\beta^a\left(1-\beta^b\right)$$

The case $Q^b = 0$ is symmetric:

$$\frac{PQ}{Y} = \frac{PQ^a}{Y} = \Lambda\beta^a - (1-\Lambda)\beta^b(1-\beta^a)$$

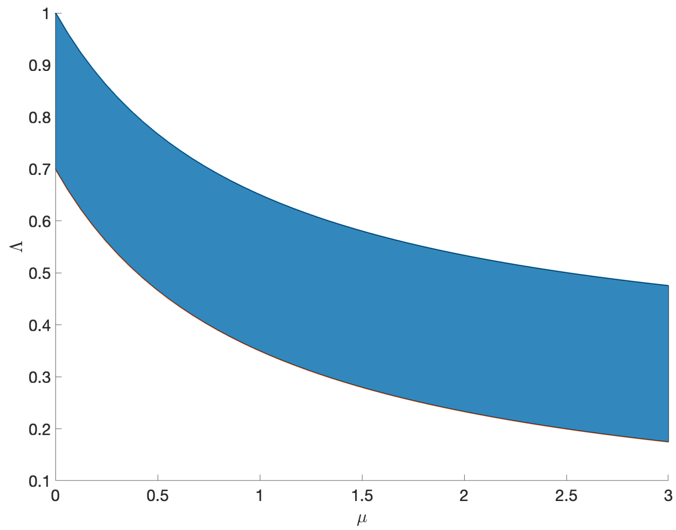

Figure 3

4.1.3 Partial efficiency

Finally, partial efficiency models have two types of equilibria. Interior solutions lead to the fully efficient outcome. Corner solutions, on the other hand, obtain when either

$$c^a + w^a L^a = \frac{1-\beta^a}{1+\mu} Y > Y^a \text{ or } \; c^b + w^b L^b = \mu \frac{1-\beta^b}{1+\mu} Y > Y^b$$

or equivalently when either

$$\frac{1-\beta^a}{1+\mu} > \frac{Y^a}{Y} = \Lambda \text{ or } \mu \frac{1-\beta^b}{1+\mu} > \frac{Y^b}{Y} \Rightarrow \Lambda > 1 - \mu \frac{1-\beta^b}{1+\mu}$$

In practice, interior solutions are in the shaded area of Figure 3.

Regarding corner solutions, in the first case, a's consumption and leisure are the same

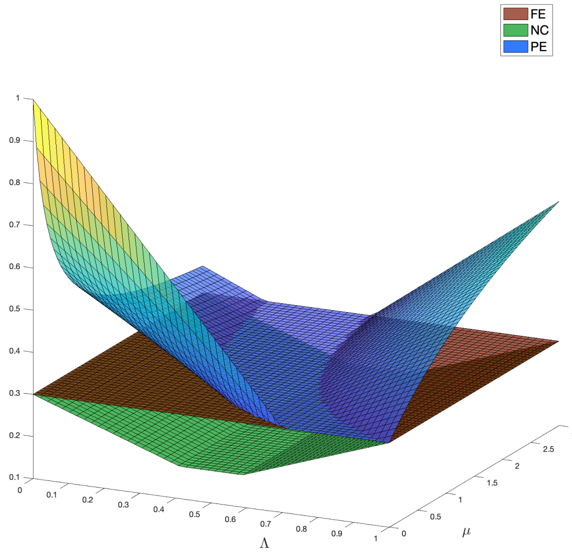

Figure 4

as in the non cooperative case, whereas b's choices are

$$w^b L^b = \frac{\mu\alpha^b}{\mu+\beta^a} Y^b, c^b = \frac{\mu\left(1-\alpha^b-\beta^b\right)}{\mu+\beta^a} Y^b$$

$$\text{and } \frac{PQ}{Y} = \frac{\beta^a+\mu\beta^b}{\mu+\beta^a}(1-\Lambda)$$

whereas in the second case

$$\frac{PQ}{Y} = \frac{\beta^a+\mu\beta^b}{1+\mu\beta^b}\Lambda$$

Figure 4 gives the level of public expenditures for FE and PE, as functions of the Pareto weight μ and of the income ratio Λ (we take $\beta^a = \beta^b = .3$). When they don't coincide, then PE corresponds to more public expenditures than FE.

4.2 Domestic production

We now consider the case of domestic production. To sharpen our analysis, we disregard monetary contributions to public good production, and we assume that the production function is:

$$\ln Q = \frac{1}{2}\left(\ln t^a + \ln t^b\right)$$

which satisfies the Constant Return to Scale assumption. Note that, in that case, all solutions to both the NC and PE cases will be corner solutions, in the sense defined above. Lastly, the global budget constraint is now

$$c^a + c^b + w^a\left(L^a + t^a\right) + w^b\left(L^b + t^b\right) = Y^a + Y^b = Y \tag{19}$$

for $Y^i = w^i T + y^i$ where T denotes total time available and y^i denotes i's non labor income (including possibly divorce settlements).

4.2.1 Full efficiency

In the FE benchmark, spouses solve:

$$\max \alpha^a \ln L^a + \left(1-\alpha^a-\beta^a\right)\ln c^a + \mu\left(\alpha^b \ln L^b + \left(1-\alpha^b-\beta^b\right)\ln c^b\right) + \frac{1}{2}\left(\beta^a+\mu\beta^b\right)\left(\ln t^a + \ln t^b\right) \tag{20}$$

under (19). Therefore

$$\begin{aligned} c^a &= \frac{\left(1-\alpha^a-\beta^a\right)Y}{1+\mu}, \quad c^b = \frac{\mu\left(1-\alpha^b-\beta^b\right)Y}{1+\mu} \\ w^a L^a &= \frac{\alpha^a Y}{1+\mu}, \quad w^b L^b = \frac{\mu\alpha^b Y}{1+\mu} \text{ and } \; w^a t^a = w^b t^b = \frac{1}{2}\frac{\left(\beta^a+\mu\beta^b\right)Y}{1+\mu} \\ \text{hence } \frac{Q}{Y} &= \frac{1}{2\sqrt{w^a w^b}}\frac{\beta^a+\mu\beta^b}{1+\mu} \end{aligned} \tag{21}$$

and market labor supply

$$w^a l^a = w^a T - \frac{\alpha^a+\beta^a+\beta^b\mu}{\mu+1}Y, \quad w^b l^b = w^b T - \frac{\beta^a+\alpha^b\mu+\beta^b\mu}{\mu+1}Y$$

In particular, comparative statics are as expected (and as discussed above). Increasing the Pareto weigth μ of individual b may either increase both t^a and t^b or decrease both, depending on the signe of $\beta^b - \beta^a$. The interpretation is straightforward: the demand for public good increases if and only if the individual whose weight has increased is the one who 'cares more' about the public good - which, in a Cobb-Douglas framework, simply means that, after proper normalization, the coefficient of public expenditures in the individual's utility is larger. Importantly, t^a and t^b always change *in the same direction*. That is, if the demand for the public good increases, then both t^a and t^b increases, and their ratio remains constant (and equal to the wage ratio).

Moreover, increasing b's Pareto weight reduces a's leisure and increases b's - reflecting the fact that leisure is a normal good. Finally, if b cares more about the public good than a does (i.e. if $\beta^b \geq \beta^a$), then increasing μ reduces b's market labor supply; while a's market labor increases b's if the leisure coefficients (the αs) are large enough and/or the βs are not too different.

Finally, we have that:

$$\frac{\partial t^a}{\partial \beta^a} = \frac{1}{2w^a}\frac{Y}{1+\mu}, \frac{\partial t^b}{\partial \beta^a} = \frac{1}{2w^b}\frac{Y}{1+\mu} \text{ and}$$
$$\frac{\partial Q}{\partial \beta^a} = \frac{1}{2\sqrt{w^a w^b}}\frac{Y}{1+\mu}$$

A decrease in β^a, a's preferences for the public good, reduces both domestic time (in proportion to individual wages), therefore total domestic production.

4.2.2 Partial efficiency

Under PE, ex spouses each solve a maximization under budget constraint. Specifically, a solves

$$\max \alpha^a \ln L^a + (1-\alpha^a-\beta^a)\ln c^a + \frac{1}{2}\left(\beta^a + \mu\beta^b\right)\ln t^a$$

under

$$c^a + w^a\left(L^a + t^a\right) = Y^a$$

while b's program is:

$$\max \mu\left(\alpha^b \ln L^b + \left(1 - \alpha^b - \beta^b\right)\ln c^b\right) + \frac{1}{2}\left(\beta^a + \mu\beta^b\right)\ln t^b$$

under

$$c^b + w^b\left(L^b + t^b\right) = Y^b$$

These give:

$$\begin{gathered}
c^a = \frac{1-\alpha^a-\beta^a}{1+\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^a, w^aL^a = \frac{\alpha^a}{1+\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^a, w^at^a = \frac{\beta^a+\mu\beta^b}{2+\left(\mu\beta^b-\beta^a\right)}Y^a \quad \text{and} \\
c^b = \frac{\mu\left(1-\alpha^b-\beta^b\right)}{\mu-\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^b, w^bL^b = \frac{\mu\alpha^b}{\mu-\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^b, w^bt^b = \frac{\beta^a+\mu\beta^b}{2\mu-\left(\mu\beta^b-\beta^a\right)}Y^b
\end{gathered} \tag{22}$$

so that

$$\begin{aligned}
\ln Q &= \frac{1}{2}\left(\ln t^a + \ln t^b\right) \\
&= \frac{1}{2}\left(\ln \frac{\beta^a+\mu\beta^b}{2+\left(\mu\beta^b-\beta^a\right)}\frac{Y^a}{w^a} + \ln \frac{\beta^a+\mu\beta^b}{2\mu-\left(\mu\beta^b-\beta^a\right)}\frac{Y^b}{w^b}\right) \\
&= \frac{1}{2}\left(\ln \frac{\beta^a+\mu\beta^b}{2+\left(\mu\beta^b-\beta^a\right)} + \ln \frac{\beta^a+\mu\beta^b}{2\mu-\left(\mu\beta^b-\beta^a\right)} + \ln\frac{Y^a}{w^a} + \ln\frac{Y^b}{w^b}\right)
\end{aligned}$$

Therefore

$$\ln\frac{Q}{Y} = \frac{1}{2}\left(\ln\Lambda + \ln\left(1-\Lambda\right)\right) - \frac{1}{2}\left(\ln w^a + \ln w^b\right) + K'$$

where

$$K' = \ln\left(\beta^a+\mu\beta^b\right) - \frac{1}{2}\ln\left(2+\left(\mu\beta^b-\beta^a\right)\right) - \frac{1}{2}\ln\left(2\mu-\left(\mu\beta^b-\beta^a\right)\right)$$

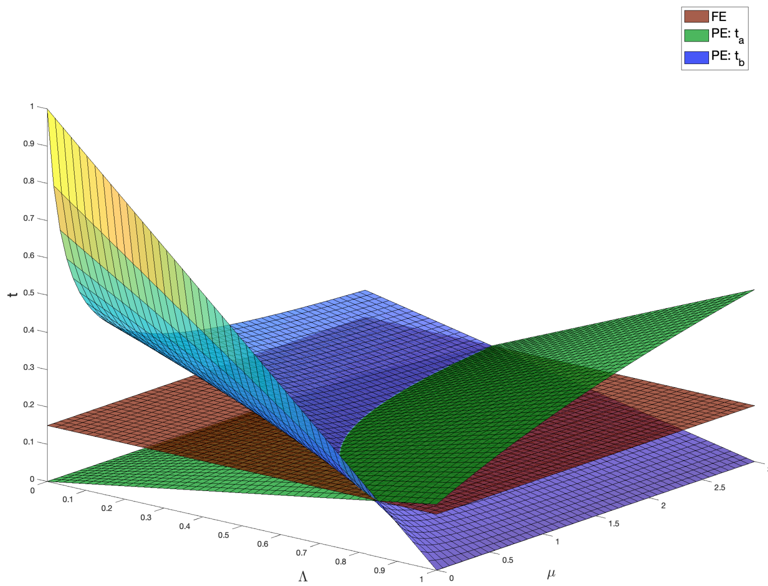

Figure 5: Domestic time

In particular, we now have that:

$$\frac{t^a}{t^b} = \frac{w^b}{w^a}\frac{Y^a}{Y^b}\frac{2\mu - \left(\mu\beta^b - \beta^a\right)}{2 + \left(\mu\beta^b - \beta^a\right)} \tag{23}$$

and the ratio t^a/t^b now depends on both the Pareto weight and the post-divorce allocation of income.

Several remarks should be made at that point. First, a direct consequence of the previous points is that the allocation of domestic time is quite different from the FE case. Figure 5 gives domestic time for FE and PE, as functions of the Pareto weight μ and of the income ratio Λ; again, we take $\beta^a = \beta^b = .3$, and moreover $w^a = w^b = Y = 1$. Interestingly, it is almost always the case that one domestic labor supply exceeds the efficient amount (which, in this formulation, is contant); then the other labor supply is below the efficient level.

Figure 2 gives the resulting level of public good production. Partial efficiency almost never coincides with full efficiency; but public good production under PE may either exceed or fall short of the optimal level. When post-divorce total incomes are similar (i.e., Λ is close to .5) and μ is unbalanced (either very small of very large), the efficient allocation would require one of the ex-spouses (the one favored by a higher Pareto weight) to reach a level of private consumption that is significantly larger than what their budget constraint allows under PE; as a result, their private consumption is suboptimal, leading to higher public consumption. In other cases, however, productive inefficiencies kick in, resulting in suboptimal levels of production.

Lastly, we can consider changes in preferences (say for a)

$$\frac{\partial t^a}{\partial \beta^a} = 2\frac{Y^a}{w^a}\frac{\beta^b\mu + 1}{\left(\beta^b\mu - \beta^a + 2\right)^2}, \frac{\partial t^b}{\partial \beta^a} = 2\frac{Y^b}{w^b}\frac{\mu\left(1-\beta^b\right)}{\left(\beta^a + 2\mu - \beta^b\mu\right)^2} \text{ and}$$

$$\frac{\partial \ln Q}{\partial \beta^a} = \frac{\beta^b\mu + 1}{2x + \left(\beta^b\right)^2\mu^2 + 2\beta^b\mu - \left(\beta^a\right)^2} + \mu\frac{1-\beta^b}{2\beta^a\mu + 2\beta^b\mu^2 - \left(\beta^a\right)^2\mu^2 + \left(\beta^a\right)^2}$$

4.2.3 Non cooperative

Finally, non cooperative behavior solves the programs:

$$\max \alpha^i \ln L^i + \left(1 - \alpha^i - \beta^i\right)\ln c^i + \frac{\beta^i}{2}\ln t^i$$

under

$$c^i + w^i\left(L^i + t^i\right) = Y^i$$

for $i = a, b$. Solutions are:

$$w^iL^i = \frac{2\alpha^i}{2-\beta^i}Y^i, w^it^i = \frac{\beta^i}{2-\beta^i}Y^i, c^i = \frac{2\left(1-\alpha^i-\beta^i\right)}{2-\beta^i}Y^i$$

and

$$\frac{Q}{Y} = \frac{\sqrt{t^at^b}}{Y} = \frac{1}{\sqrt{w^aw^b}}\sqrt{\frac{\beta^a\beta^b}{\left(2-\beta^a\right)\left(2-\beta^b\right)}}\sqrt{\frac{Y^a}{Y}}\sqrt{\frac{Y^b}{Y}}$$

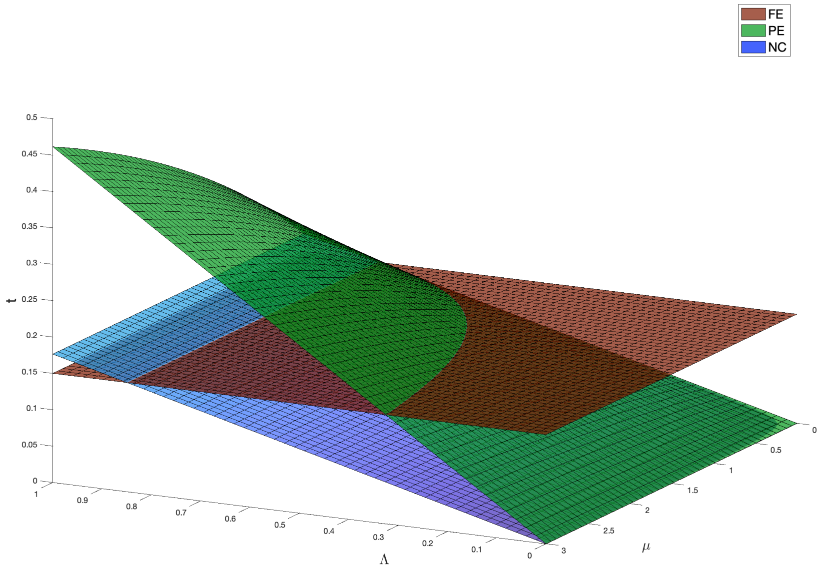

Figure 6: Spouse a's domestic time under FE, PE and NC

Figure 2 gives the resulting level of public production under the three regimes. As expected, the public good production is always (largely) suboptimal under NC, due to suboptimal investment by both parents. In particular, a comparison of domestic times between the three regimes is provided by Figure 6.

Lastly,

$$\frac{\partial t^a}{\partial \beta^a} = \frac{Y^a}{w^a} \frac{2}{\left(2-\beta^a\right)^2}, \frac{\partial t^b}{\partial \beta^a} = 0 \text{ and}$$

$$\frac{\partial Q}{\partial \beta^a} = \sqrt{\frac{Y^a Y^b}{w^a w^b}} \frac{1}{2-\beta^a} \sqrt{\frac{\beta^b}{\beta^a}} \frac{1}{\sqrt{\left(2-\beta^b\right)\left(2-\beta^a\right)}}$$

In particular, an individual's domestic time does not respond to changes in the partner's preferences for the public good.