Divorce with children: the partial efficiency approach

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6 February 2025

Abstract

We introduce a new concept called *partial efficiency* (PE) to model the post-divorce behaviors of ex-spouses. We assume that divorced parents still care about their children and maintain an efficient approach to the provision of the public good, but they do not share risk or compensate each other's private consumption. We show that the PE approach offers more realistic implications than the full-efficiency (FE) or non-cooperative (NC) models of divorce.

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1 Introduction

The analysis of divorce and its welfare implications has recently attracted renewed attention from economists.[1] Not only is individual behavior after divorce (and its consequences in terms of poverty and inequality) interesting per se, but it may play a crucial role in understanding behavior when married, if only because it provides a possible threat point for any bargaining model of the married couple. Existing models have focused on how divorce outcomes affect behavior during marriage but have not focused on post-divorce interactions, imposing the simplifying assumption that following separation ex-spouses stop interacting with each other and go their separate ways (Voena, 2015; Fernández and Wong, 2017; Low, Meghir, Pistaferri *et al.*, 2022). Such an assumption, however, ignores a basic fact, namely that in most cases both divorced parents still care about their children and contribute to their welfare. In fact, a host of crucially important issues, such as the impact of divorce on child development, precisely depend on understanding how post-divorce interactions and decisions take place.

The goal of this note is to discuss the various modeling options available to explicitly analyze such a situation. We will focus on investments in child human capital, which we take to be a public good that the ex-spouses still enjoy following separation. We first argue that neither a standard collective model (Chiappori, 1988, 1992; Blundell, Chiappori, and Meghir, 2005) nor a non-cooperative approach provide an adequate framework for understanding these interactions. The collective model relies on an efficiency assumption, particularly in terms of risk sharing and private consumption, that contradicts the potentially adversarial nature of the relationship between ex-spouses. Non cooperative models, on the other hand, involve private provision of the public good, which has been known to deliver largely counterfactual predictions. We require a modelling framework that is capable of fitting the existing data patterns and to provide a way of understanding the underlying

[1] For some key contributions see Fernández and Wong (2017); Lafortune and Low (2017); Wolfers (2006); Stevenson (2007, 2008); Stevenson and Wolfers (2006); Voena (2015); Rasul (2005).

mechanisms that drive investments and ultimately outcomes for children of divorced couples. This is particularly important given the need to understand why children of divorced parents appear to have worse outcomes, which could be a combination of selection effects and changes in parental investments.

We introduce a new concept called *partial efficiency*. In a partially efficient context, individuals maintain an efficient approach to the provision of the public good; in particular, their contribution to public expenditures takes into account the benefits derived by the ex-spouse. Regarding all other economic decisions, however, individuals fail to cooperate: they do not share risk or compensate each other's private consumptions. We show that the partial efficiency approach differs from both the fully cooperative and the non cooperative framework, with distinct empirical implications. For example, contrary to the concept of partial efficiency that we will now introduce, the non-cooperative model implies income pooling when both ex-partners are contributing to the public good, while the cooperative model implies full insurance between them even post divorce. Both these implications can be rejected with post-divorce data on private and public expenditures.

2 The model with no domestic production

We consider a couple consuming four commodities: two individual leisures, L^a and L^b, a public good Q (that could be interpreted, depending on the context, as expenditures on children, children's human capital or more globally as children's welfare), and a Hicksian private good, c^a and c^b, which are generally not separately observed. We first consider the case of purely monetary contributions towards the public good; domestic production will be analyzed in the next section.

Individual preferences are summarized by a utility of the form $U^i\left(c^i, L^i, Q\right)$ for spouse $i = a, b$. When married, we assume individuals reach Pareto-efficient agreements. Their behavior is thus represented by a standard, collective setting:

$$\max_{c^a, c^b, L^a, L^b, Q} U^a\left(c^a, L^a, Q\right) + \mu U^b\left(c^b, L^b, Q\right) \tag{1}$$

under the budget constraint

$$c^a + c^b + PQ + w^a L^a + w^b L^b = \left(w^a + w^b\right) T + y \tag{2}$$

Here, w^i denotes i's wage, T is total time available, y is the couple's non labor income and μ is a Pareto weight.

We now assume that the individuals under consideration have divorced; that is, U^i now denotes post-divorce individual utilities, which may or may not be identical to pre-divorce ones. We can assume that non labor income y is then divided between the spouses - i.e., individual i receives y^i, with $y^a + y^b = y$. Note that y^i may be negative (then $y^j, j \neq i$, exceeds y); that would be the case, for instance, if the divorce settlement involves part of i's labor income be transferred to j. Crucially, commodity Q remains public after divorce; that is, both ex-spouses still care about children's welfare, although *how much* they do may be affected by divorce.

In order to model the ex-spouses' decision process, we successively consider three possible settings.

2.1 Full efficiency

A first approach would assume that individuals still reach a fully efficient (from now on FE) agreement. Then they jointly solve a problem similar to (1)-(2); the only difference with the pre-divorce situation is that individual utilities and the Pareto weight may have changed after divorce. Note, in particular, that they face a unique, common budget constraint, though they have separate incomes. In practice, thus, the ex-spouses jointly maximize (1) under

$$c^a + w^a L^a + c^b + w^b L^b + PQ = Y^a + Y^b$$

and the time allocation constraints $0 \leq L^i \leq T$ for $i = a, b$. In the above, Y^i denotes i's potential income:

$$Y^i = w^i T + y^i, \quad i = a, b.$$

An efficient decision can always be represented as stemming from a two-stage process. In stage 1, individuals jointly choose the total quantity of the public good Q and a *sharing rule* (ρ^a, ρ^b) that defines how the remaining resources are split between them. In stage 2, they independently decide on their labor supply and private consumption, under the budget constraint defined by the sharing rule. The second stage decision of individual $i = a, b$ solves:

$$V^i\left(w^i, \rho^i, Q\right) = \max_{c^i, L^i} U^i\left(c^i, L^i, Q\right) \text{ under } c^i + w^i L^i = \rho^i$$

The function V^i is the conditional indirect utility of i.[2] The first stage program is:

$$\max_{Q, \rho^a, \rho^b} V^a\left(w^a, \rho^a, Q\right) + \mu V^b\left(w^b, \rho^b, Q\right) \text{ under } \rho^a + \rho^b + PQ = Y = Y^a + Y^b,$$

and the first order conditions give the usual, Bowen-Lindahl-Samuelson equations:

$$MWP^a + MWP^b = \frac{\partial V^a / \partial Q}{\partial V^a / \partial \rho^a} + \frac{\partial V^b / \partial Q}{\partial V^b / \partial \rho^b} = P$$

expressing the fact that individual marginal willingness to pay (MWP) for the public good

[2] In the particular case of separable preferences:

$$U^i\left(c^i, L^i, Q\right) = \bar{U}^i\left(u^i\left(c^i, L^i\right), Q\right)$$

the first stage only affects individual decisions in stage 2 through the budget constraint. In general, however, the marginal rate of substitution (MRS) between leisure and private consumption are also affected by public expenditures; in that sense, the indirect utility is defined conditionally on Q.

add up to its price.

This solution requires a level of cooperation that may not be realistic for divorced individuals. Assume, in particular, that the solution to the previous program, $\left(\bar{c}^a, \bar{L}^a, \bar{c}^b, \bar{L}^b, \bar{Q}\right)$, is such that

$$\bar{c}^i + w^i \bar{L}^i > w^i T + y^i \quad \text{for some } i.$$

Then the program implies that the private consumptions of spouse i exceeds their total resources, hence involving transfers from j to i that exceed those mandated by the divorce settlement (the latter being subsumed in the y^is). Under FE, such transfers are fully implementable.

2.2 Non cooperation

Alternatively, we may assume that ex-spouses choose not to cooperate. Since public good expenditures still enter both utilities, implying that they both may want to contribute to it, the corresponding game is a private contribution one in which ex-spouses each choose their contribution taking the other spouse's as given.

Formally, in the non cooperative (henceforth NC) model ex-spouse a (similarly for b) solves the program

$$\max_{c^a, L^a, Q} U^a\left(c^a, L^a, Q\right) \tag{3}$$

under

$$\begin{aligned} c^a + w^a L^a + PQ &= Y^a + PQ^b \\ Q &\geq Q^b \end{aligned} \tag{4}$$

and the time allocation constraint $0 \leq L^a \leq T$. In the above, a chooses how much to spend on the public good (Q^a) taking as given b's expenditure (Q^b), so $Q = Q^a + Q^b$. The problems of a and b are simultaneously solved, so that the expectations of ex-spouses about each other hold true in equilibrium.

The properties of the corresponding Nash equilibria are well-known (Bergstrom, Blume, and Varian, 1986; Browning, Chiappori, and Lechene, 2009). Assuming interior labour supply choices for simplicity, the first order conditions for the problems of the two ex-spouses give the set of equations:

$$\frac{\partial U^a/\partial L^a}{\partial U^a/\partial c^a} = w^a; \quad \frac{\partial U^b/\partial L^b}{\partial U^b/\partial c^b} = w^b; \quad \frac{\partial U^a/\partial Q}{\partial U^a/\partial c^a} = MWP^a = P; \quad \frac{\partial U^b/\partial Q}{\partial U^b/\partial c^b} = MWP^b = P$$

In general, three of the above conditions (the first three or, alternatively, the first two and the last) together with the two budget constraints (as in (4)) are sufficient to determine private consumptions, leisure and public consumption (c^a, L^a, c^b, L^b, Q). This means that the equilibrium solution can take one of two forms. It will correspond to a corner solution when the fourth condition is violated, whereby the public good is entirely funded by one spouse.[3] In practice, that would imply that one ex-spouse (usually the custodial parent) takes in charge the full amount of children expenditures, with no help from the other parent.

Alternatively, the solution may be interior, in the sense that both individuals contribute to the funding (and the fourth condition holds). In that case, the resulting equilibrium exhibits a strong *income pooling* property. Namely, the allocation, including the distribution of private consumption and leisure among ex-spouses, only depends on total (ex-) household resources, not on its distribution between ex-spouses. In particular, a change in the divorce settlement - say, an increase in y^a and a corresponding decrease in y^b - cannot impact individual behavior or well-being. Differently from the FE case, where the bargaining weights may respond to changes in the income distribution and impact allocations, the NC model has no natural way of capturing such effects except in the extreme case of only one ex-spouse contributing to the public good. Such conclusion appears to be largely counterfactual.

[3]This happens when either $\frac{\partial U^a/\partial Q}{\partial V^a/\partial c^a} < P$, in which case only b contributes to the public good, or $\frac{\partial U^b/\partial Q}{\partial V^b/\partial c^b} < P$, in which case only a contributes.

2.3 Partial efficiency

We now introduce a third setting which borrows from the previous two and provides, in our view, a more nuanced model of choice after divorce. Just like FE, partial efficiency (from now on PE) can be seen as the outcome of a two-stage process. In stage one, ex-spouses jointly decide on the amount to be spent on the public good and on each ex-spouse's contribution. Importantly, the decision is assumed to be efficient, in the (usual) sense that it maximizes a weighted sum of individual utilities under a budget constraint. In stage two, individuals each choose their consumption and market labor supply under the constraints defined by stage one.

A crucial difference, however, is that in a FE context stage 1 determines both the quantity of the public good *and* the sharing rule; the latter will in turn determine individual (market) labor supplies and private consumptions. Under PE, stage 1 does not fix a sharing rule, but simply individual contributions to the public good - the latter being moreover subject to a non negativity constraint. The sharing rule is then determined by restricting direct transfers between spouses, so ex-spouses keep separate budget constraints as in the NC model.

In others words, in a PE context decisions on public good expenditures explicitly consider the benefits derived by *both* ex-partners - this is the efficiency part. Efficiency, however, is only partial, because *direct* transfers across couples are ruled out (beyond those implied by the divorce settlement); as a consequence, ex-spouses each face their own budget constraint, which may or may not affect efficiency.[4]

The formal translation of these ideas depends on the context, and in particular on the presence of domestic production. In models that only allow for monetary contributions towards public consumption, the two-stage solution is as follows. In stage 2, conditional on individual contributions to the public good $\left(Q^a, Q^b\right)$, where $Q = Q^a + Q^b$, individual $i = a, b$

[4]For instance, risk sharing mechanisms are strongly limited by this assumption.

solves the following program:

$$V^i\left(w^i, Y^i - PQ^i, Q\right) \;=\; \max_{c^i, L^i} U^i\left(c^i, L^i, Q\right)$$

under the budget constraint

$$c^i + w^i L^i = Y^i - PQ^i$$

and the time allocation constraint $0 \leq L^i \leq T$, where again T denotes total available time and V^i is the conditional indirect utility.

The collective decision process at stage 1 is summarized by the following program:

$$\max_{Q^a, Q^b} \; V^a\left(w^a, Y^a - PQ^a, Q^a + Q^b\right) + \mu V^b\left(w^b, Y^b - PQ^b, Q^a + Q^b\right)$$

under the constraints $Q^i \geq 0, i = a, b$.

We see, in particular, that while direct monetary transfers between ex-spouses are ruled out, the allocation of public good expenditures across ex-spouses is unconstrained. In particular, *implicit* transfers via changes in the individuals' respective contributions are possible. The only constraint is non negativity; in practice, it requires that an individual's *private* consumption (including leisure) cannot exceed the individual's potential income.

Two cases must therefore be considered. If the solution to the FE framework, $\left(\bar{c}^a, \bar{L}^a, \bar{c}^b, \bar{L}^b, \bar{Q} = \bar{Q}^a + \bar{Q}^b\right)$, is such that

$$\bar{c}^i + w^i \bar{L}^i \leq Y^i \text{ for all } i, \tag{5}$$

then one can define i's contribution $\bar{Q}^i$ by

$$P\bar{Q}^i = Y^i - \left(\bar{c}^i + w^i \bar{L}^i\right)$$

It follows that the FE solution is compatible with the PE constraints, in which case the two solutions coincide.

In the alternative case, one of the constraints - say, for a - is violated. Then a corner solution obtains. That is, a's second stage program becomes:

$$V^a\left(w^a, Y^a, Q\right) = \max_{c^a, L^a} U^a\left(c^a, L^a, Q\right) \quad \text{under} \quad c^a + w^a L^a = Y^a,$$

while the program of b is

$$V^b\left(w^b, Y^b - PQ, Q\right) = \max_{c^b, L^b} U^b\left(c^b, L^b, Q\right) \quad \text{under} \quad c^b + w^b L^b = Y^b - PQ$$

The first stage program becomes:

$$\max_Q V^a\left(w^a, Y^a, Q\right) + \mu V^b\left(w^b, Y^b - PQ, Q\right)$$

The first order conditions now imply:

$$\frac{1}{\mu}\frac{\partial V^a/\partial Q}{\partial V^b/\partial Y^b} + \frac{\partial V^b/\partial Q}{\partial V^b/\partial Y^a} = P$$

which fails to be efficient since

$$\frac{\partial V^a}{\partial Y^a} \neq \mu\frac{\partial V^b}{\partial Y^b}$$

at a corner solution. The interpretation is straightforward. The constraint (5) is now binding for $i = a$, implying that the ratio of marginal utilities of income differs from the Pareto weight. In practice, the solution is now inefficient, since a's private consumptions are less than what the FE model would imply.

Two points may be noted here:

- The PE solution typically generates more public good expenditures than the NC one.

This simply reflects the fact that, when choosing public expenditure, b also takes into account the utility a derives from the public good.

- More surprinsingly, the PE solution typically generates more public good expenditures than the FE one - at least when they differ. The intuition is that the two concepts differ when one individual constraint is binding - i.e., when the FE solution would imply more private consumption for one individual than what the individual's personnal budget constraint allows. With well-behaved utilities, this results in less private expenditures for that individual, therefore more public expenditures overall, under PE than under FE. Note, however, that this conclusion heavily depends on the absence of domestic production, as we show below.

3 Model with domestic production

We now consider the more interesting case in which the public good is produced within the household - a situation that fits the interpretation in terms of children welfare. Specifically, we now assume that

$$Q = \phi\left(t^a, t^b\right)$$

where t^i denotes i's time devoted to domestic production.[5] Empirically, the output Q is usually not directly observed, implying that the scale of the process is unobservable; only the inputs t^a and t^b are recorded (say, from time use data). Then a natural assumption is that the technology described by ϕ exhibits constant returns to scale:

$$\phi\left(kt^a, kt^b\right) = k\phi\left(t^a, t^b\right) \text{ for all } k > 0$$

[5] A more general case would involve both time and money as inputs to the production function. Then solutions can either interior or corner.

implying that

$$\phi\left(t^a, t^b\right) = t^b \psi\left(\frac{t^a}{t^b}\right) \text{ for some } \psi$$

As a consequence the partial derivatives of ϕ only depend on the ratio t^a/t^b:

$$\frac{\partial \phi}{\partial t^a} = \psi'\left(\frac{t^a}{t^b}\right), \quad \frac{\partial \phi}{\partial t^b} = \psi - \frac{t^a}{t^b}\psi'\left(\frac{t^a}{t^b}\right)$$

3.1 Full efficiency

As before, we start with the FE benchmark. Here, the ex-spouses jointly solve

$$\max_{(c^i, L^i, t^i)_{i=a,b}} U^a\left(c^a, L^a, \phi\left(t^a, t^b\right)\right) + \mu U^b\left(c^b, L^b, \phi\left(t^a, t^b\right)\right)$$

under

$$c^a + w^a L^a + c^b + w^b L^b = Y^a - w^a t^a + Y^b - w^b t^b$$

and the time constraints $L^i + t^i \leq T$ and $L^i, t^i \geq 0$ for $i = a, b$.

The first order conditions are:

$$\frac{\partial U^a}{\partial c^a} = \lambda, \quad \mu\frac{\partial U^b}{\partial c^b} = \lambda, \quad \frac{\partial U^a}{\partial L^a} = \lambda w^a, \quad \mu\frac{\partial U^b}{\partial L^b} = \lambda w^b$$

and

$$\left(\frac{\partial U^a}{\partial Q} + \mu\frac{\partial U^b}{\partial Q}\right)\frac{\partial \phi}{\partial t^a} = \lambda w^a, \quad \left(\frac{\partial U^a}{\partial Q} + \mu\frac{\partial U^b}{\partial Q}\right)\frac{\partial \phi}{\partial t^b} = \lambda w^b$$

where λ denotes the Lagrange multiplier of the budget constraint. Thus the first order conditions for the FE model with domestic production imply a specific version of the Bowen-Lindahl-Samuelson equations:

$$\left(MWP^a + MWP^b\right)\frac{\partial \phi}{\partial t^i} = w^i, \quad i = a, b$$

In particular, we have that:

$$\frac{\partial \phi / \partial t^b}{\partial \phi / \partial t^a} = \frac{w^b}{w^a}$$

This equation pins down the ratio t^a/t^b. In particular, *that ratio does not depend on* μ. Under this setting, the *scale* of public good production (represented by the level of public good production Q) depends on the intra-household power allocation: more power to the spouse who 'cares more' about the public good (in the sense defined by Blundell, Chiappori, and Meghir, 2005) results in more production of it. For a given scale, however, the allocation of inputs is totally driven by productive efficiency; it is simply the cheapest way to produce the desired level. In particular, the ratio of a and b's time contributions is not affected by the intra-household balance of power; under the CRS assumption, it is fully determined by the ratio of individual wages and the production technology.[6]

3.2 Non cooperation

We next consider the NC model, with individuals each choosing their domestic and market labor supply while taking the ex-spouse's contribution to domestic production as given. First order conditions for individual i ($i = a, b$) are

$$\frac{\partial U^i}{\partial c^i} = \lambda^i, \qquad \frac{\partial U^i}{\partial L^i} = \frac{\partial U^i}{\partial Q}\frac{\partial \phi}{\partial t^i} = \lambda^i w^i \tag{6}$$

where λ^i is i's marginal utility of income. In contrast with the NC model with purely monetary public expenditure, here the time contributions of ex-spouses generally depend on own wages, and also on the distribution of resources through the marginal value of consumption. The resulting allocations are inefficient. In particular, two conclusions emerge:

[6]This point suggests, in particular, that the spouses' respective contributions to domestic production need not reflect the distribution of power within the household; it all depends on the type of efficiency assumed to be achieved by the decision process.

- The public good is under produced; indeed, (6) implies that

$$MWP^i \frac{\partial \phi}{\partial t^i} = w^i, \quad i = a, b \tag{7}$$

Intuitively, individuals each fail to consider the benefit their investment will provide to their spouse.

- This (insufficient) amount of public good is *inefficiently* produced, in the sense that the same level of public good could be produced at a lower total cost (thus potentially allowing more private leisure and consumption for both spouses). Indeed, (7) implies that

$$\frac{\partial \phi / \partial t^b}{\partial \phi / \partial t^a} = \frac{w^b}{w^a} \times \frac{MWP^a}{MWP^b} \neq \frac{w^b}{w^a}$$

In words, if $MWP^a > MWP^b$, then a invests too much (and b too little) time into public good production, as compared to what productive efficiency would require.

3.3 Partial efficiency

Finally, the PE solution can again be obtained from a two stage procedure. In stage 2, individual i solves

$$V^i \left(w^i, Y^i - w^i t^i, \phi \left(t^a, t^b\right)\right) = \max_{c^i, L^i} U^i \left(c^i, L^i, \phi \left(t^a, t^b\right)\right)$$

under $0 \leq L^i \leq T - t^i$ and

$$c^i + w^i L^i = Y^i - w^i t^i$$

where V^i is again i's conditional indirect utility. The first stage is therefore

$$\max_{t^a, t^b} V^a \left(w^a, Y^a - w^a t^a, \phi \left(t^a, t^b\right)\right) + \mu V^b \left(w^b, Y^b - w^b t^b, \phi \left(t^a, t^b\right)\right)$$

under $0 \leq t^i \leq T$, which gives the following first order conditions:

$$\begin{aligned}
\left(\frac{\partial V^a}{\partial Q}+\mu\frac{\partial V^b}{\partial Q}\right)\frac{\partial\phi}{\partial t^a} &= w^a\frac{\partial V^a}{\partial Y^a} \text{ and} \\
\left(\frac{\partial V^a}{\partial Q}+\mu\frac{\partial V^b}{\partial Q}\right)\frac{\partial\phi}{\partial t^b} &= w^b\frac{\partial V^b}{\partial Y^b}
\end{aligned}$$

In particular,

$$\frac{\partial\phi/\partial t^b}{\partial\phi/\partial t^a} = \frac{\partial V^b/\partial Y^b}{\partial V^a/\partial Y^a}\times\frac{w^b}{w^a}\neq\frac{w^b}{w^a} \tag{8}$$

In this case, the ex-spouses time contributions depend not only on wages, but also on the Pareto weight and the post-divorce allocation of income (through the corresponding marginal utilities of income). In particular, *productive efficiency does not obtain:* for almost all outcomes, it would have been possible to achieve the same level of public good at a lower (total) cost – but that would have required transfers between spouses that are not feasible under PE. The ex-spouse whose marginal value of income is larger under PE (i.e., in the absence of direct transfers) than FE will supply less domestic labour than would be efficient, while the other ex-spouse (partly) compensates. The production level of public goods, however, could be either lower or higher than the efficient level, as illustrated by the following example.

4 A Cobb-Douglas example

We now illustrate the previous results by considering specific preferences. Namely, assume that utilities are Cobb-Douglas:

$$U^i\left(c^i,L^i,Q\right)=\alpha^i\ln L^i+\beta^i\ln Q+\left(1-\alpha^i-\beta^i\right)\ln c^i$$

and so the weighted sum of utilities is also Cobb-Douglas:

$$U^a + \mu U^b = \alpha^a \ln L^a + (1 - \alpha^a - \beta^a) \ln c^a + \mu\alpha^b \ln L^b + \mu \left(1 - \alpha^b - \beta^b\right) \ln c^b + \left(\beta^a + \mu\beta^b\right) \ln Q$$

In what follows, we let Y denote total household income, and Λ the fraction coming from a:

$$\Lambda = Y^a/Y$$
$$\text{where } Y^i = w^i T + y^i \text{ for } i = a, b.$$

Moreover, the baseline scenario assumes that preferences are homogeneous with parameters $\beta^a = \beta^b = 0.3$.

4.1 Basic version: no domestic production

We start with the basic model; precise derivations of all results can be found in the Online Appendix. The global budget constraint is

$$c^a + c^b + w^a L^a + w^b L^b + PQ = Y^a + Y^b = Y$$

Figure 1 shows the level of public expenditures Q as a fraction of total income Y under FE, NC and PE, as functions of the Pareto weight μ and of the income ratio Λ. Given homogeneous preferences, public good expenditure is independent of both μ and Λ under FE. As expected, NC reduces public good expenditures in comparison to both FE and PE. The gap is larger when the income distribution is more balanced and the solution is interior (flat part of the curve).

FE and PE obtain the same solution in the part of the space where total optimal private expenditure for each individual is such that no transfers are required between the

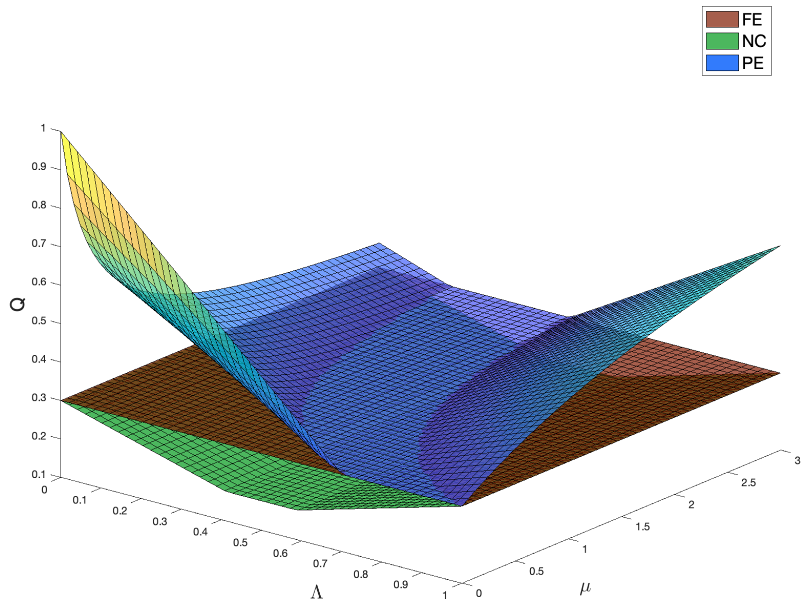

Figure 1: Ratio of public good expenditure to total income as a function of a's income share (Λ) and b's Pareto weight (μ)

ex-spouses, i.e.:

$$c^a + w^a L^a = \frac{1-\beta^a}{1+\mu} Y \leq Y^a \quad \text{and} \quad c^b + w^b L^b = \mu \frac{1-\beta^b}{1+\mu} Y \leq Y^b$$

However, if transfers are required to implement the first best under FE (i.e., when either $\frac{1-\beta^a}{1+\mu} Y > Y^a$ or $\mu \frac{1-\beta^b}{1+\mu} Y > Y^b$), PE (where such transfers are not feasible) results in more public good expenditures than FE, as discussed above. This counterintuitive result is dependent on public goods being simply purchased in the market and may not hold once we introduce domestic production. Under PE, it is also interesting that public good expenditure increases with b's Pareto weight when b is comparatively poor, and decreases when b is comparatively rich. In other words, more bargaining power will enable b to extract more utility by either: (i) inducing a to spend more in the public good when b's resources are low, or (ii) reducing public expenditure, hence keeping more of their own income for private consumption and leisure, when their resources are high.

4.2 Domestic production

We now consider the case of domestic production. To keep things simple, we assume that the production function is also Cobb-Douglas and depends only on time inputs (t^a, t^b):

$$\ln Q = \frac{1}{2}\left(\ln t^a + \ln t^b\right)$$

which imposes constant return to scale.The global budget constraint is now:

$$c^a + c^b + w^a\left(L^a + t^a\right) + w^b\left(L^b + t^b\right) = Y^a + Y^b = Y.$$

As before, the derivations are left for the Appendix and we state here the main results. With FE, the ratio of time inputs only depends on the wage ratio:

$$\frac{t^a}{t^b} = \frac{w^b}{w^a}$$

Under PE that ratio deviates from productive efficiency and depends on the Pareto weight and preferences, as well as the wage ratio:

$$\frac{t^a}{t^b} = \frac{w^b}{w^a}\frac{2\mu - \left(\mu\beta^b - \beta^a\right)}{2 + \left(\mu\beta^b - \beta^a\right)}\frac{Y^a}{Y^b}$$

Finally, NC behavior leads to the ratio:

$$\frac{t^a}{t^b} = \frac{w^b}{w^a}\frac{\beta^a\left(2 - \beta^b\right)}{\beta^b\left(2 - \beta^a\right)}\frac{Y^a}{Y^b}$$

Under FE with constant returns to scale, the ratio of time inputs to the production of public good is determined by (and, in the Cobb-Douglas case, equal to) the ratio of their market wage, and does not depend on preference parameters or the Pareto weights. This is a consequence of the production efficiency property of the FE equilibrium.

Once we depart from FE, and move either to a NC equilibrium or to our new concept of PE, the inputs get distorted away from efficient provision. This can for instance lead to one of the partners providing more than the efficient benchmark while the other provides less. This outcome depends on the configuration of individual incomes, preferences and (under PE) relative bargaining power. The overall provision of the public good will be different from the efficient outcome, but whether it is higher or lower will depend on the parameter configuration.

4.3 The comparative statics of divorce

Divorce may affect the choices of partners through various mechanisms. Firstly, under full efficiency both before and after marriage, the Pareto weights may change; moreover, preferences towards the public good may also be affected, for instance in the case of children if the non-custodial parent becomes more detached after divorce. In addition, and still under the assumption of FE in marriage, agreements upon divorce may switch to PE or NC.

We now investigate the empirical implications of such changes. For brevity, we concentrate on the domestic production model, and we stick to Cobb-Douglas preferences and production technology, although most conclusion are valid more generally. Table 1 summarises expected changes in time dedicated to domestic production upon divorce, and in total public good consumption. The direction of changes reported in the Table are discussed in the next sections.

Table 1: Predicted responses in domestic production upon divorce

	Change upon divorce	Regime in divorce FE	PE	NC
(1)	Regime (from FE in marriage)	t^a : unchanged t^b : unchanged Q : unchanged	(t^a, t^b): move in opposite directions $Q : \downarrow$ (most cases)	$t^a : \downarrow$ (most cases) $t^b : \downarrow$ (most cases) $Q : \downarrow$
(2)	Increase in PW μ (spouse who most values Q)	$t^a : \uparrow$ $t^b : \uparrow$ $Q : \uparrow$	$t^a : \uparrow$ $t^b : \downarrow$ Q : undetermined	- - -
(3)	Drop in β^a (preference for Q of one spouse)	$t^a : \downarrow$ $t^b : \downarrow$ $Q : \downarrow$	$t^a : \downarrow$ $t^b : \downarrow$ $Q : \downarrow$	$t^a : \downarrow$ t^b : unchanged $Q : \downarrow$

Notes: Row (1) shows differences in time investments in children and child outcomes between married and divorced families, only the regime changes but all preference parameters are the same. Row (2) shows how changing the Pareto weight of one spouse affects the allocations and outcomes, conditional on the regime. Row (3) shows how changing preferences for child outcomes of one spouse affects allocations and outcomes, conditional on the regime.

4.3.1 Changing regime: from full efficiency to partial efficiency or non cooperation

As discussed above, productive efficiency is generally not achieved under PE. For the Cobb-Douglas specification, one domestic labor supply almost always exceeds the efficient amount while the other is below the efficient level (see figure 4 in the Online Appendix).

Figure 2 shows the level of public good production for the three regimes (FE, PE and NC), by the Pareto weight μ and the fraction Λ of total income going to ex-spouse a. PE almost never coincides with FE, but public good production under PE may either exceed or fall short of the optimal level. When post-divorce total incomes are similar and μ is unbalanced (either very small or very large), the efficient allocation would require one of the ex-spouses (the one favored by a higher Pareto weight) to reach a level of private consumption that is significantly larger than what their budget constraint allows under PE. As a result, and just as in the no production case, the private consumption of that individual

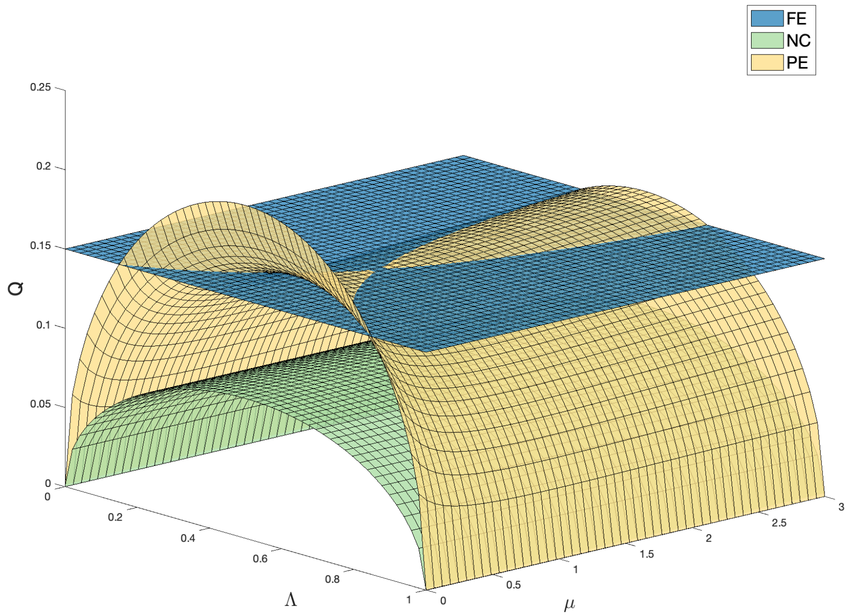

Figure 2: Public good provision with domestic production under FE, PE and NC

is suboptimal, leading to higher public consumption. In most cases, however, productive inefficiencies kick in and dominate, resulting in *suboptimal* levels of production. Finally, the public good production is always (largely) suboptimal under NC, due to low investments by both parents.

4.3.2 Changes in Pareto weights

We now consider a change in Pareto weights in either a FE or PE context post-divorce. Keeping a's weight normalized to 1, suppose b's weight μ increases. Under both FE and PE, b's leisure and private consumption increase, while a's decrease. Regarding market work, it decreases for b and increases for a under PE. The FE case is more complex; yet, if one assumes that preferences for public good are 'not too different' - technically, if

$$\left|\beta^a - \beta^b\right| < \min\left(\alpha^a, \alpha^b\right),$$

then the same conclusion obtains.

The main difference is related to domestic time. Under FE, changes are totally driven by preferences for the public good. If b 'cares more' about the public good than a does[7] - that is, in the Cobb-Douglas case, if $\beta^b > \beta^a$ - then more power to b tends to increase the total production of public good. As a result, *both domestic times increase* (while their ratio remains constant). If $\beta^b < \beta^a$, the opposite conclusion obtains. Importantly, t^a and t^b always change *in the same direction.*

The PE case is different: when μ gets larger, a's domestic time always increases while b's always decreases, and the impact on total domestic production is ambiguous. In sharp contrast with the FE case, when Pareto weights change under PE then individual domestic times always move in *opposite* directions.

4.3.3 Changes in preferences

Finally, what are the consequences of a decrease in a spouse's (say, a's) preferences for the public good? For example, in the case where the human capital of the child is the public good, the distance between the non-custodial parent and the child may reduce their attachment, which we capture here by a reduction in the utility weight of the public good in the non-custodial's utility function. A technical issue is how the decrease affects the MRS between consumption and leisure. In what follows, we simply consider a decrease in β^a that keeps α^a constant (thus increasing preferences for consumption); the main qualitative conclusions are unaffected. The main implications are as follows.[8]

- Under FE, both domestic times decrease, and their ratio remain equal to the wage ratio. Intuitively, decreasing β^a while keeping β^b constant reduces the total weight of the public good in the household's maximization program, resulting in a lower production level; productive efficiency then requires that individual changes be inversely

[7] Again in the sense of Blundell, Chiappori, and Meghir (2005).
[8] See the Online Appendix for more detail.

proportional to wages

- Under PE, the total weight of the public good also decreases, which again leads to a reduction in both domestic times. However, the ratio of domestic times is also affected (specifically, t^a/t^b increases); moreover, the impact also depends on post-divorce allocations.

- Finally, under NC, a's domestic time is reduced; in the Cobb-Douglas case, b's is unaffected.

4.4 Identifiability

We use the Panel Study of Income Dynamics (PSID) linked with the Child Development Supplements (CDS) to study parents' time investments in children. The CDS Time Diary records all child activities on a random weekday and a weekend, and is available in three waves in 1997, 2002 and 2007. We compute domestic time inputs as the total time that the mother or father is actively participating or engaged with the child, including educational and recreational activities as well as cleaning, feeding, dressing the child etc. In Table 2 we document changes in domestic time and labor supply after divorce.

Table 2: Changes in domestic time and labor supply after divorce

	Domestic time input		Labor supply	
	Father (1)	Mother (2)	Father (3)	Mother (4)
Divorced × Education of (Father, Mother)				
Divorced × (Low, Low)	-4.555***	-1.829***	.0113	.0955***
	(.4283)	(.5157)	(.0157)	(.0224)
Divorced × (Low, High)	-4.392***	.5299	-.0322	-.0294
	(.7847)	(.8718)	(.0287)	(.0379)
Divorced × (High, Low)	-3.433***	-4.729***	-.0197	.1645**
	(.9631)	(1.214)	(.0352)	(.0528)
Divorced × (High, High)	-7.883***	.4366	.0106	.0505
	(.9638)	(1.161)	(.0353)	(.0505)
N	4446	4998	4446	4998

NOTES: Domestic time is the total time actively engaged with a child (hours per week). Labor supply is an indicator for working or not. All four columns include controls for child age dummy, full interactions between father and mother's education types, and an indicator for those who are ever divorced. Columns (1) and (2) also control for hours of work and an indicator for working. "High" refers to college degree or more; "Low" refers to some college or less. Standard errors in parentheses. * 0.05 ** 0.01 *** 0.001

A key question is whether the distinction between FE, NC and PE has empirical content, and specifically whether FE and NC can be rejected. Consider the patterns of change around divorce as documented in Table 2. These patterns of change seem more complex than can be explained by FE. For instance, productive efficiency in the domestic production process typically requires similar directions of changes in time inputs for mothers and fathers, which is contradicted by the data. It is also unlikely that the NC model fits these patterns, since in most cases it would predict that both mother's and father's inputs decline. It is tempting to take these patterns as a rejection of FE or NC. However, this would be misleading because divorce can also lead to changes in preferences and in the production function for child human capital, which we take to be a public good.

Based on the results in Chiappori (1988, 1992) and Blundell, Chiappori, and Meghir (2005), we know the conditions under which we can identify preferences and the sharing rule in the marriage state. We can also identify the production function of human capital separately for married and divorced couples. Finally, among divorced couples we observe the consumption of the private goods, which allows us to identify preferences for the mother and the father. Given these it is possible to test whether FE, PE or NC hold.

5 Concluding Remarks: The consequences of divorce - what do we expect?

For understanding the effects of divorce on children and even more so the mechanisms that underlie these effects, we need a modelling framework than can fit the complex changes in behavior when a couple separates. In this short note we have outlined three alternative approaches to modelling behavior and we have argued that PE is the most flexible approach. Neither the FE nor the NC outcomes can, single-handedly, capture the richness of responses to divorce that we observe in the data.

Divorce is a complex phenomenon, which may affect the couple's behavior in several ways. It may change individual respective incomes and bargaining powers, which in turn influence the spouses' allocation of time between leisure, domestic and market work. It might also modify preferences, for instance by decreasing the value put on public consumption by one of the ex-partners (typically the non custodial parent). Finally, it is likely to change the nature of the intra-household decision process away from efficiency. Interestingly, the various alternative assumptions about post-divorce behavior lead to different empirical predictions on responses. Specifically:

- The FE model has very clear-cut implications. Whether the changes affect preferences or Pareto weights, if FE prevails both before and after divorce, then both domestic times should always move in the same direction; under constant returns, their ratio should moreover remain constant (and determined by the wage ratio). In particular, the gender asymmetry in behavior often observed after divorce (whereby, in many households, the father's domestic time decreases while the mother's increases or remains constant) seems incompatible with this model.

- If, alternatively, divorce leads to a switch from FE to NC, one should expect a large decrease in both partners' domestic labor supplies. Again, this prediction does not seem consistent with observed empirical patterns. In addition, when both individuals contribute financially to the public good, their choices should be invariant to the distribution of income, another counterfactual prediction.

- A more flexible approach involves a switch from FE to PE. This regime change always increases one domestic time while decreasing the other, the impact on both total time and production level being ambiguous. If, in addition, preferences change as well, the decrease in one ex-partner's time will be exacerbated, whereas the increase in the ex-spouse's time will be mitigated (but will not necessarily disappear).

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Online Appendix: Cobb-Douglas preferences

We assume that individual preferences are Cobb-Douglas:

$$U^i(c^i, L^i, Q) \;=\; \alpha^i \ln L^i + \left(1 - \alpha^i - \beta^i\right) \ln c^a + \beta^i \ln Q \qquad \text{for } i = a, b$$

where (c^i, L^i) are the consumption and leisure of partner i, and total public expenditure is Q. In the above (α^i, β^i) are parameters. In the illustration, we set $\beta^a = \beta^b = 0.3$.

The total resources of each partner are $Y^i = w^i T + y^i$, where T is total time, w is the wage rate and y is other income, which could include divorce settlements. Total household resources are $Y = Y^a + Y^b$. In what follows, we let Λ denote the ratio of a's income to total income:

$$\Lambda \;=\; Y^a / Y$$

A Basic version: no domestic production

A.1 Full efficiency

In the full efficiency benchmark, the program is

$$\max_{c^a, c^b, L^a, L^b, Q} U^a(c^a, L^a, Q) + \mu U^b(c^b, L^b, Q) \tag{9}$$

$$\text{s.t} \;\; c^a + c^b + w^a L^a + w^b L^b + PQ \;=\; Y^a + Y^b \;=\; Y \tag{10}$$

under time constraints $0 \leq L^a, L^b \leq T$. The interior solution is:

$$\begin{aligned} c^a &= \frac{1 - \alpha^a - \beta^a}{1 + \mu} Y \quad \text{and} \quad w^a L^a \;=\; \frac{\alpha^a}{1 + \mu} Y, \\ c^b &= \frac{\mu \left(1 - \alpha^b - \beta^b\right)}{1 + \mu} Y \quad \text{and} \quad w^b L^b \;=\; \frac{\mu \alpha^b}{1 + \mu} Y, \\ PQ &= \frac{\beta^a + \mu \beta^b}{1 + \mu} Y. \end{aligned}$$

A.2 Non cooperative

The program of ex-spouse a (and similarly to b) in the NC case is

$$\max_{c^a,L^a,Q^a} U^a(c^a, L^a, Q) \tag{11}$$

$$\text{s.t} \quad c^a + w^a L^a + PQ^a = Y^a \tag{12}$$

under the time and expenditure constraints, $0 \leq L^a, L^b \leq T$ and $Q^q, Q^b \geq 0$, and with $Q = Q^a + Q^b$.

The interior solution (where both partners contribute to the public good) is:

$$PQ^a = \frac{\beta^a Y^a - (1-\beta^a)\,\beta^b Y^b}{1-(1-\beta^a)\,(1-\beta^b)} \quad \text{and} \quad PQ^b = \frac{\beta^b Y^b - \left(1-\beta^b\right)\beta^a Y^a}{1-(1-\beta^a)\,(1-\beta^b)},$$

and therefore

$$PQ = \frac{\beta^a \beta^b}{1-(1-\beta^a)\,(1-\beta^b)} Y.$$

Moreover:

$$c^i = \frac{(1-\alpha^i-\beta^i)\beta^j}{1-(1-\beta^a)(1-\beta^b)} Y \quad \text{and} \quad w^i L^i = \frac{\alpha^i \beta^j}{1-(1-\beta^a)(1-\beta^b)} Y$$

for $i, j = a, b$, with $i \neq j$. As expected, the solution satisfies income pooling, as it only depends on total income Y, not on individual incomes Y^a and Y^b.

Corner solutions, on the other hand, obtain when the previous solution is such that either

$$\begin{aligned} Y^a\beta^a - Y^b\beta^b\,(1-\beta^a) \leq 0 \quad &\Rightarrow \quad \Lambda \leq \frac{\beta^b\,(1-\beta^a)}{\beta^a + \beta^b\,(1-\beta^a)} \\ \text{or } Y^b\beta^b - Y^a\beta^a\left(1-\beta^b\right) \leq 0 \quad &\Rightarrow \quad \Lambda \geq \frac{\beta^b}{\beta^a\,(1-\beta^b) + \beta^b} \end{aligned}$$

In the first case, $Q^a = 0$, then

$$\begin{aligned}
w^a L^a &= \frac{\alpha^a}{1-\beta^a} Y^a \quad \text{and} \quad c^a = \frac{1-\alpha^a-\beta^a}{1-\beta^a} Y^a, \\
w^b L^b &= \alpha^b Y^b \quad \text{and} \quad c^b = \left(1-\alpha^b-\beta^b\right) Y^b, \\
\frac{PQ}{Y} &= \frac{PQ^b}{Y} = (1-\Lambda)\,\beta^b - \Lambda\beta^a\left(1-\beta^b\right)
\end{aligned}$$

The case $Q^b = 0$ is symmetric:

$$\frac{PQ}{Y} = \frac{PQ^a}{Y} = \Lambda\beta^a - (1-\Lambda)\,\beta^b\,(1-\beta^a)$$

A.3 Partial efficiency

The PE program is

$$\max_{c^a,c^b,L^a,L^b,Q^a,Q^b} U^a(c^a, L^a, Q) + \mu U^b(c^b, L^b, Q) \tag{13}$$

$$\text{s.t} \quad c^i + w^i L^i + PQ^i = Y^i \text{ for } i = a, b \tag{14}$$

under the time and expenditure constraints, $0 \leq L^a, L^b \leq T$ and $Q^q, Q^b \geq 0$, and with $Q = Q^a + Q^b$.

PE models have two types of equilibria. Interior solutions lead to the FE outcome. Corner solutions, on the other hand, obtain when either

$$c^a + w^a L^a = \frac{1-\beta^a}{1+\mu} Y > Y^a \quad \text{or} \quad c^b + w^b L^b = \mu\frac{1-\beta^b}{1+\mu} Y > Y^b$$

or, equivalently, when either

$$\frac{1-\beta^a}{1+\mu} > \Lambda \quad \text{or} \quad \mu\frac{1-\beta^b}{1+\mu} > 1-\Lambda$$

In our illustration, interior solutions are in the shaded area of Figure 3, by (μ, Λ). In points

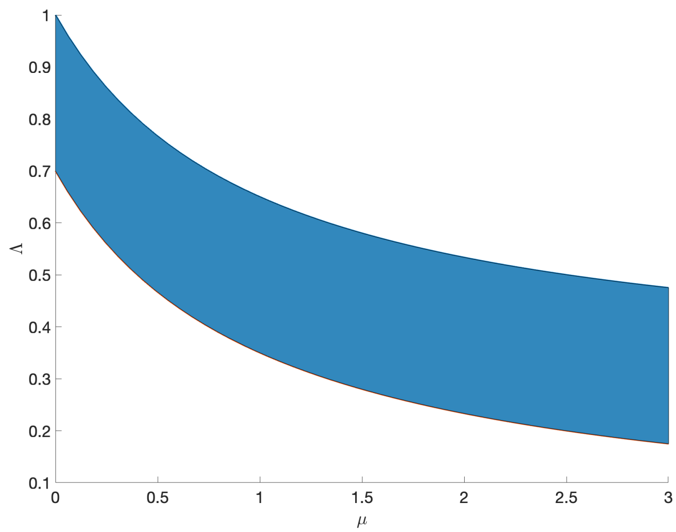

Figure 3: Space of interior solutions under PE, by the Pareto weight μ and a's fraction of household resources Λ

under the shaded area, only b contributes to the public good, while only a contributes in points above it. The figure shows that when bargaining power is especially high for one of the spouses (e.g. μ close to zero, with a detaining most of the negotiating power), that spouse will only contribute to the public good if they also hold most of the income. With more balanced power (around $\mu = 1$), both spouses contribute when inequality in income is low, but with high inequality only the better-off spouse contributes.

Regarding corner solutions, in the first case a's consumption and leisure are the same as in the non cooperative case, whereas b's choices are

$$\begin{aligned} w^b L^b &= \frac{\mu \alpha^b}{\mu + \beta^a} Y^b, \quad c^b = \frac{\mu \left(1 - \alpha^b - \beta^b\right)}{\mu + \beta^a} Y^b, \\ \text{and } \frac{PQ}{Y} &= \frac{\beta^a + \mu \beta^b}{\mu + \beta^a} (1 - \Lambda) \end{aligned}$$

whereas in the second case

$$\frac{PQ}{Y} = \frac{\beta^a + \mu \beta^b}{1 + \mu \beta^b} \Lambda$$

B Domestic production

We now consider the case of domestic production, assuming for simplicity that it depends only on the time spent by each spouse on producing the public good, hence disregarding monetary contributions:

$$\ln Q = \frac{1}{2}\left(\ln t^a + \ln t^b\right) \tag{15}$$

where (t^a, t^b) are time spent in domestic production. This production function satisfies constant return to scale. Note that, in that case, all solutions to both the NC and PE cases will be corner solutions, in the sense defined above.

B.1 Full efficiency

In the FE benchmark, the family solves the problem

$$\max_{(c^i,L^i,t^i)_{i=a,b}} \quad U^a(c^a, L^a, Q) + \mu U^b(c^b, L^b, Q) \tag{16}$$

$$\text{s.t.} \quad c^a + c^b + w^a\left(L^a + t^a\right) + w^b\left(L^b + t^b\right) \;=\; Y^a + Y^b \;=\; Y \tag{17}$$

under the production function (15) and the time constraints $L^i + t^i \leq T$ and $L^i, t^i \geq 0$ for $i = a, b$.

The solution is

$$\begin{aligned}
c^a &= \frac{1-\alpha^a-\beta^a}{1+\mu}Y, \quad \text{and} \quad w^a L^a = \frac{\alpha^a}{1+\mu}Y, \\
c^b &= \frac{\mu\left(1-\alpha^b-\beta^b\right)}{1+\mu}Y \quad \text{and} \quad w^b L^b = \frac{\mu\alpha^b}{1+\mu}Y, \\
w^a t^a &= w^b t^b = \frac{1}{2}\frac{\beta^a+\mu\beta^b}{1+\mu}Y \quad \text{and hence} \quad \frac{Q}{Y} = \frac{1}{2\sqrt{w^a w^b}}\frac{\beta^a+\mu\beta^b}{1+\mu}
\end{aligned} \tag{18}$$

resulting in market labor supplies:

$$w^a l^a \;=\; w^a T - \frac{\alpha^a+\beta^a+\beta^b\mu}{\mu+1}Y \quad \text{and} \quad w^b l^b \;=\; w^b T - \frac{\beta^a+\alpha^b\mu+\beta^b\mu}{\mu+1}Y \tag{19}$$

The above expressions (18) and (19) can be used straightforwardly to determine how choices vary with the Pareto weight μ. To examine how the production of public good varies with preferences for it (the β's), it is useful to write:

$$\frac{\partial t^a}{\partial \beta^a} = \frac{1}{2w^a}\frac{Y}{1+\mu}, \quad \frac{\partial t^b}{\partial \beta^a} = \frac{1}{2w^b}\frac{Y}{1+\mu} \quad \text{and} \quad \frac{\partial Q}{\partial \beta^a} = \frac{1}{2\sqrt{w^a w^b}}\frac{Y}{1+\mu}. \tag{20}$$

B.2 Partial efficiency

Under PE, ex spouses maximise the objective function (16) under the production function (15), the separate budget constraints

$$c^i + w^i\left(L^i + t^i\right) = Y^i \quad \text{for } i = a, b,$$

and the time constraints as above.

The solution is:

$$c^a = \frac{1-\alpha^a-\beta^a}{1+\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^a, \quad w^a L^a = \frac{\alpha^a}{1+\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^a, \quad w^a t^a = \frac{\beta^a+\mu\beta^b}{2+\left(\mu\beta^b-\beta^a\right)}Y^a$$

and

$$c^b = \frac{\mu\left(1-\alpha^b-\beta^b\right)}{\mu-\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^b, \quad w^b L^b = \frac{\mu\alpha^b}{\mu-\frac{1}{2}\left(\mu\beta^b-\beta^a\right)}Y^b, \quad w^b t^b = \frac{\beta^a+\mu\beta^b}{2\mu-\left(\mu\beta^b-\beta^a\right)}Y^b$$

so that

$$\begin{aligned}\ln Q &= \frac{1}{2}\left(\ln t^a + \ln t^b\right)\\ &= \frac{1}{2}\left(\ln\frac{\beta^a+\mu\beta^b}{2+\left(\mu\beta^b-\beta^a\right)} + \ln\frac{\beta^a+\mu\beta^b}{2\mu-\left(\mu\beta^b-\beta^a\right)} + \ln\frac{Y^a}{w^a} + \ln\frac{Y^b}{w^b}\right)\end{aligned}$$

Therefore

$$\ln\frac{Q}{Y} = \frac{1}{2}\left(\ln\Lambda + \ln\left(1-\Lambda\right)\right) - \frac{1}{2}\left(\ln w^a + \ln w^b\right) + K'$$

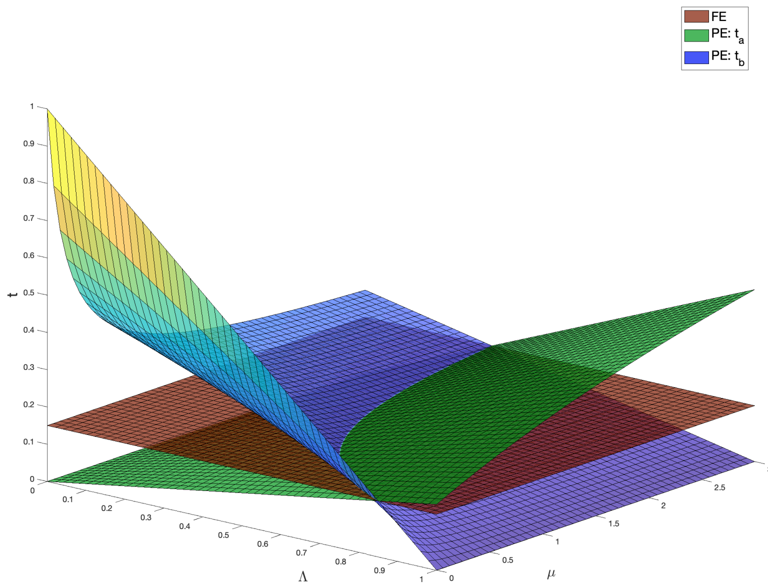

Figure 4: Domestic time

where

$$K' = \ln\left(\beta^a + \mu\beta^b\right) - \frac{1}{2}\ln\left(2 + \left(\mu\beta^b - \beta^a\right)\right) - \frac{1}{2}\ln\left(2\mu - \left(\mu\beta^b - \beta^a\right)\right)$$

In particular, we now have:

$$\frac{t^a}{t^b} = \frac{w^b}{w^a}\frac{Y^a}{Y^b}\frac{2\mu - \left(\mu\beta^b - \beta^a\right)}{2 + \left(\mu\beta^b - \beta^a\right)}$$

which shows that the ratio t^a/t^b now depends on both the Pareto weight and the post-divorce allocation of income.

A direct consequence of the previous points is that the allocation of domestic time is quite different from the FE case. Figure 4 gives domestic time for FE and PE, as functions of the Pareto weight μ and of the income ratio Λ (we take parameters $\beta^a = \beta^b = .3$ as above, and set $w^a = w^b = Y^a = Y^b = 1$). As predicted, it is almost always the case that one domestic labor supply exceeds the efficient amount (which, in this formulation, is constant) while the other labor supply is below the efficient level.

To inspect how the solution varies with the preferences for the public good, we can consider changes in β^a:

$$\begin{aligned}\frac{\partial t^a}{\partial \beta^a} &= 2\frac{Y^a}{w^a}\frac{\beta^b\mu+1}{\left(\beta^b\mu-\beta^a+2\right)^2}, \qquad \frac{\partial t^b}{\partial \beta^a} = 2\frac{Y^b}{w^b}\frac{\mu\left(1-\beta^b\right)}{\left(\beta^a+2\mu-\beta^b\mu\right)^2} \qquad \text{and}\\ \frac{\partial \ln Q}{\partial \beta^a} &= \frac{\beta^b\mu+1}{2x+\left(\beta^b\right)^2\mu^2+2\beta^b\mu-\left(\beta^a\right)^2}+\mu\frac{1-\beta^b}{2\beta^a\mu+2\beta^b\mu^2-\left(\beta^a\right)^2\mu^2+\left(\beta^a\right)^2}\end{aligned}$$

B.3 Non cooperative

Finally, in a NC agreement each spouse $i=a,b$ solves the program:

$$\begin{aligned}&\max_{c^i,L^i,t^i} U(c^i,L^i,Q)\\ &\text{s.t. } c^i+w^i\left(L^i+t^i\right) = Y^i\end{aligned}$$

under the production function (15) and the time constraints described before. The solutions are:

$$\begin{aligned}w^iL^i &= \frac{2\alpha^i}{2-\beta^i}Y^i, \quad w^it^i = \frac{\beta^i}{2-\beta^i}Y^i, \quad c^i = \frac{2\left(1-\alpha^i-\beta^i\right)}{2-\beta^i}Y^i \quad \text{and}\\ \frac{Q}{Y} &= \frac{\sqrt{t^at^b}}{Y} = \frac{1}{\sqrt{w^aw^b}}\sqrt{\frac{\beta^a\beta^b}{\left(2-\beta^a\right)\left(2-\beta^b\right)}}\sqrt{\frac{Y^a}{Y}}\sqrt{\frac{Y^b}{Y}}\end{aligned}$$

As discussed in the main text, the public good production is always (largely) suboptimal under NC, due to suboptimal investment by both parents. In particular, a comparison of domestic times between the three regimes is provided by Figure 5.

Domestic production changes with preference parameter β^a as follows:

$$\begin{aligned}\frac{\partial t^a}{\partial \beta^a} &= \frac{Y^a}{w^a}\frac{2}{\left(2-\beta^a\right)^2}, \qquad \frac{\partial t^b}{\partial \beta^a} = 0 \qquad \text{and}\\ \frac{\partial Q}{\partial \beta^a} &= \sqrt{\frac{Y^aY^b}{w^aw^b}}\frac{1}{2-\beta^a}\sqrt{\frac{\beta^b}{\beta^a}}\frac{1}{\sqrt{\left(2-\beta^b\right)\left(2-\beta^a\right)}}\end{aligned}$$

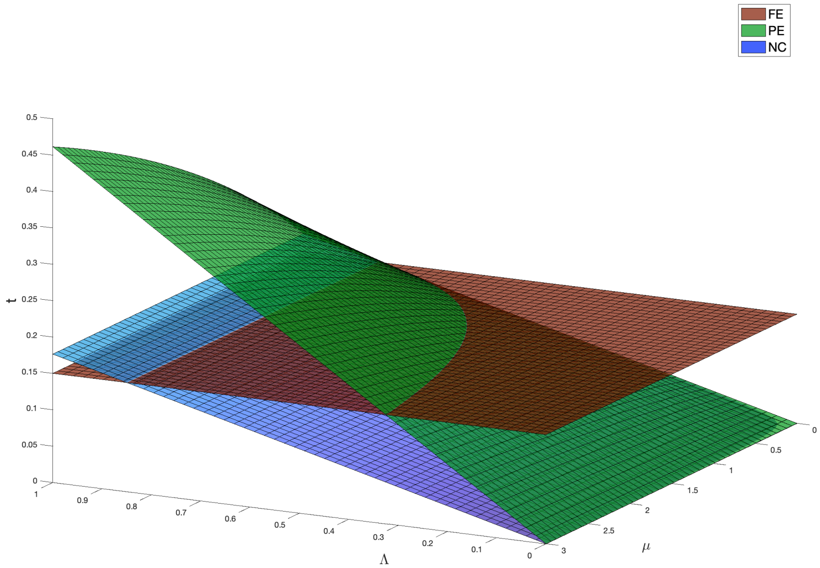

Figure 5: Spouse a's domestic time under FE, PE and NC

In this case, an individual's domestic time does not respond to changes in the partner's preferences for the public good.